永磁同步馬達電流迴路設計/驗證及功率量測

The current regulator design/verification and power measurement of permanent magnet synchronous motors (PMSM)

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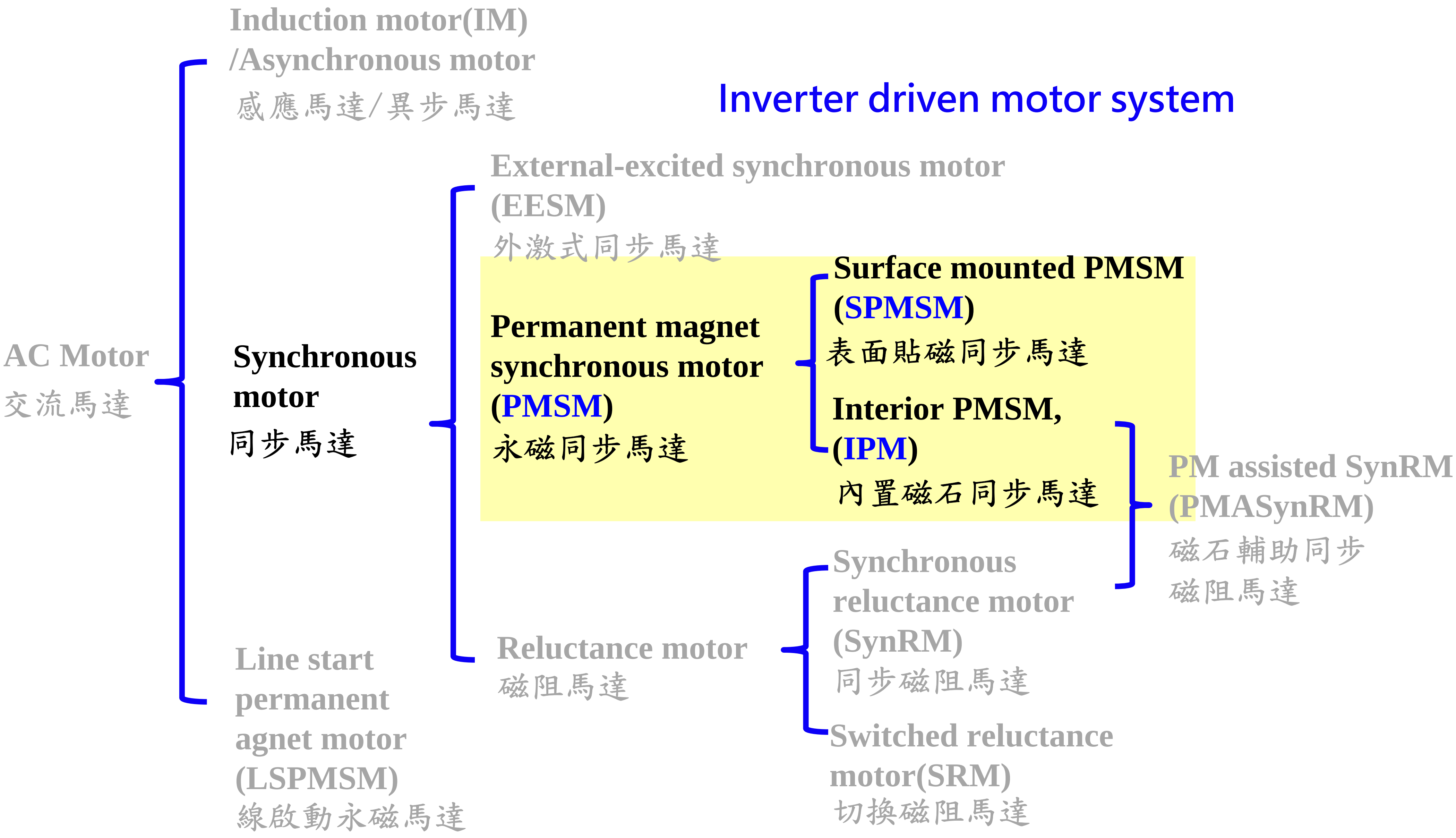
時間　：2022年4月28日

大綱

1. 控制架構
2. 電流迴路設計
3. 基於波德圖之電流迴路設計範例 - PI 補償器
4. 電流迴路之驗證
5. 以示波器量測電氣功率及相關訊號
6. 結論

控制架構 – 永磁同步馬達



Inverter driven motor system

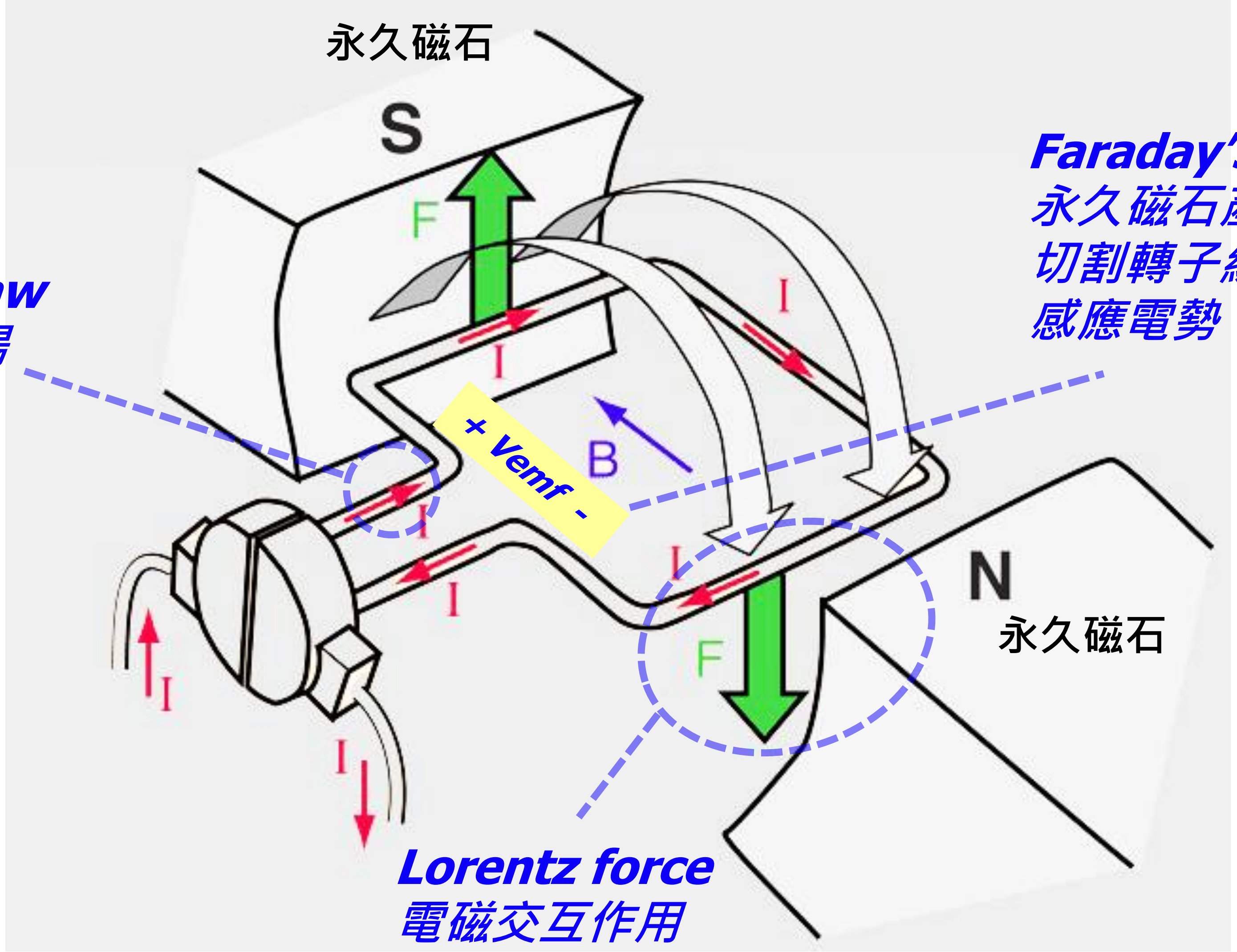
- AC Motor 交流馬達
  - Induction motor(IM) /Asynchronous motor 感應馬達/異步馬達
  - Synchronous motor 同步馬達
    - External-excited synchronous motor (EESM) 外激式同步馬達
    - Permanent magnet synchronous motor (**PMSM**) 永磁同步馬達
      - Surface mounted PMSM (**SPMSM**) 表面貼磁同步馬達
      - Interior PMSM, (**IPM**) 內置磁石同步馬達
    - Reluctance motor 磁阻馬達
      - Synchronous reluctance motor (SynRM) 同步磁阻馬達
      - Switched reluctance motor(SRM) 切換磁阻馬達
  - Line start permanent agnet motor (LSPMSM) 線啟動永磁馬達

PM assisted SynRM (PMASynRM) 磁石輔助同步磁阻馬達

控制架構 – 馬達產生輸出轉矩之原理

□關鍵定理(以直流馬達結構說明)

- ✓ **Faraday’s law**
- ✓ **Ampere’s law**
- ✓ **Lorentz force**



Ampere’s law
電流產生磁場
B=f(I)

Faraday’s law
永久磁石產生磁場
切割轉子線圈造成
感應電勢

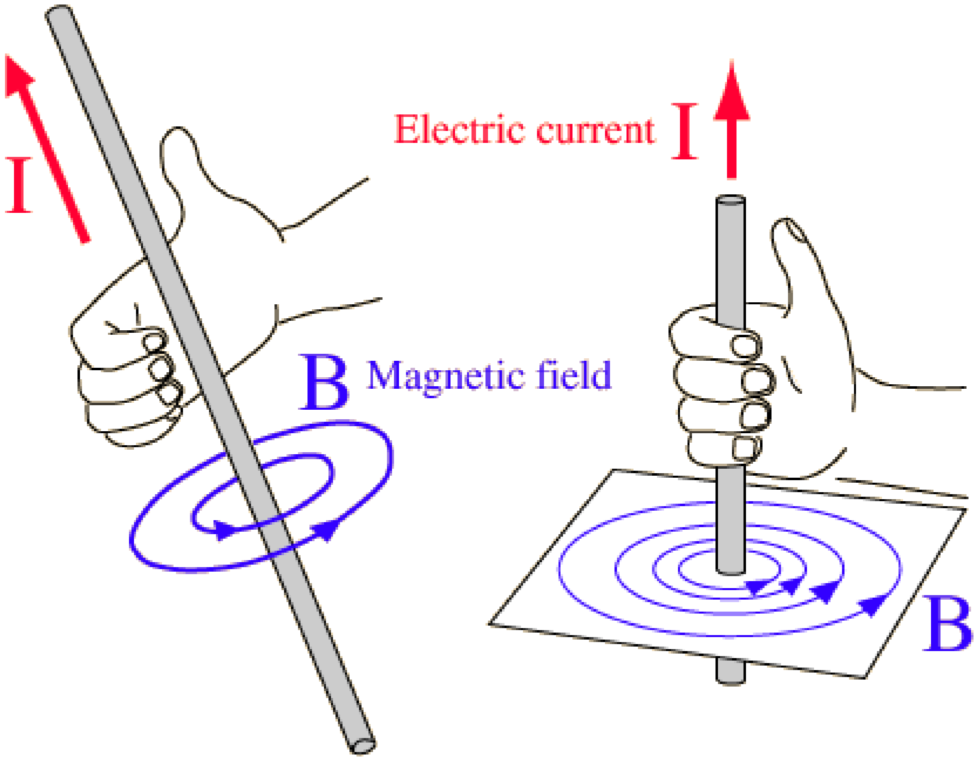
Lorentz force
電磁交互作用
F=ILB

http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motdc.html

控制架構 – 永磁同步馬達控制原理

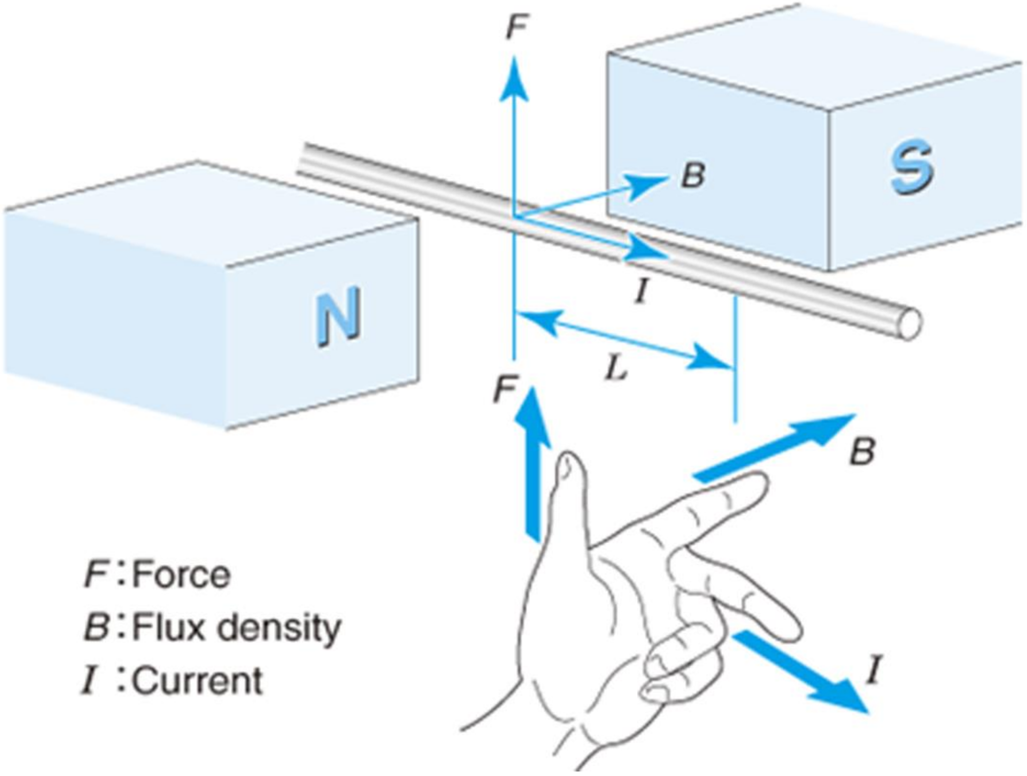
□關鍵定理

✓Ampere’s law
電流流經導線產生磁場



✓Lorentz force
電磁交互作用產生力

(Fleming's Left Hand Rule)



https://www.nidec.com/cn/technology/motor/basic/00012/

控制架構 – 永磁同步馬達控制原理

□輸出轉矩(*T*)

$$T = F \times r = (I\ \ L\ \ B) \times r$$

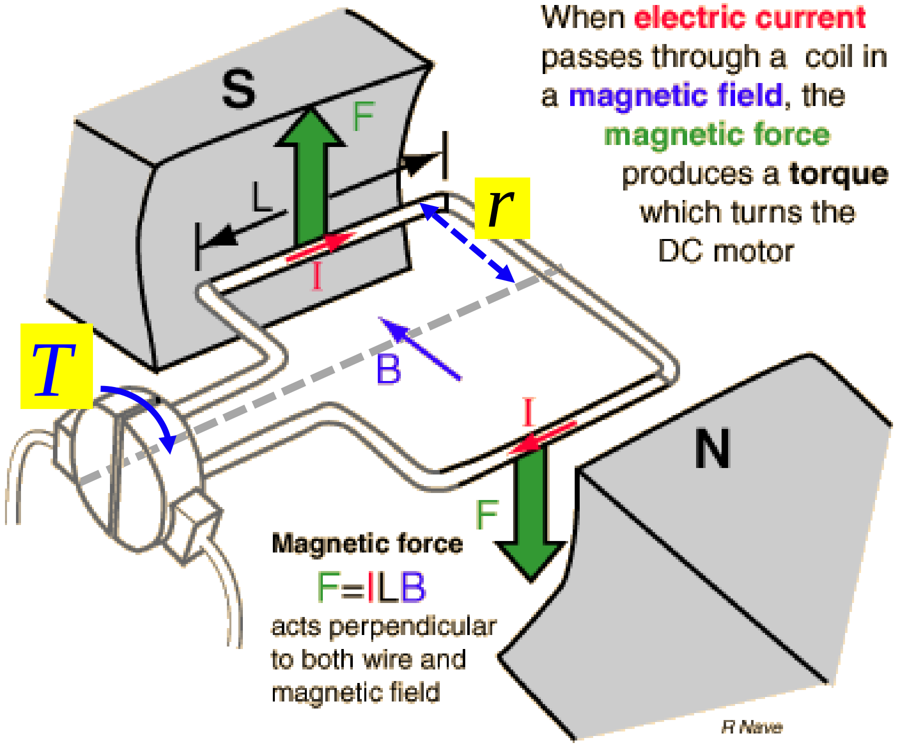
- I：流經導線電流
- L：導線與磁通密度(B)交鏈的長度
- B：由永久磁石產生之磁通密度
- r：導線L至中心點距離

✓當電流與磁場正交時會得到最大之輸出轉矩

✓若B固定不變，T與I成正比

✓T與r成正比

✓磁通密度(B)是電、機能量轉換之媒介



When **electric current** passes through a coil in a **magnetic field**, the **magnetic force** produces a **torque** which turns the DC motor

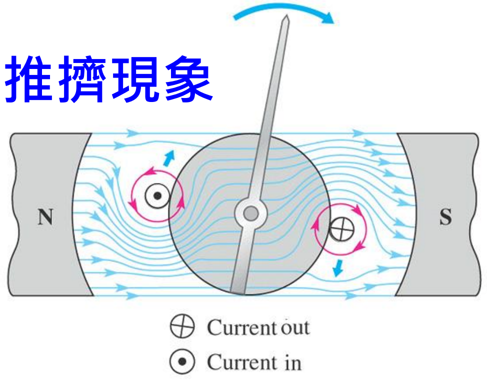
Magnetic force

F=ILB

acts perpendicular to both wire and magnetic field

http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motdc.html

磁力線推擠現象

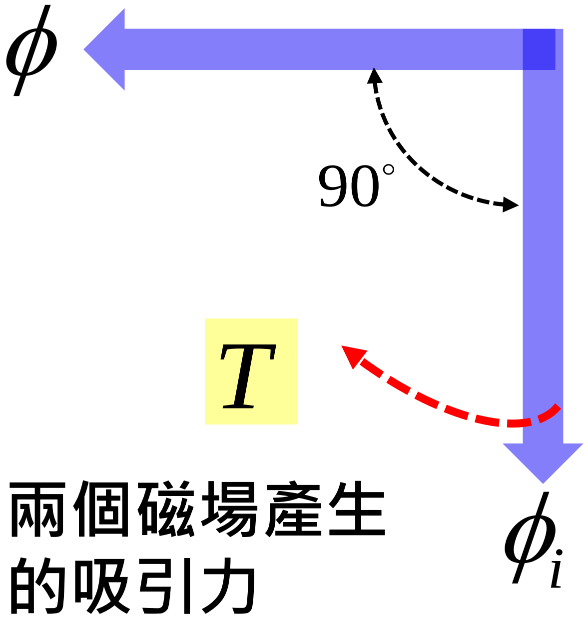


控制架構 – 永磁同步馬達控制原理

□**輸出轉矩(*T*)** ➡ 定子與轉子磁場交互作用產生的輸出轉矩

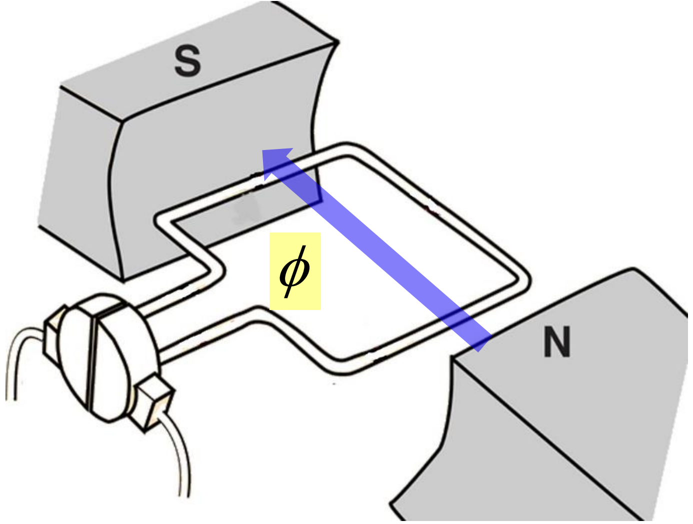
$$T=F\times r=(BLI)\times r=k_1 B_i(I)B$$

$$T=K\,\phi_i\,\phi$$

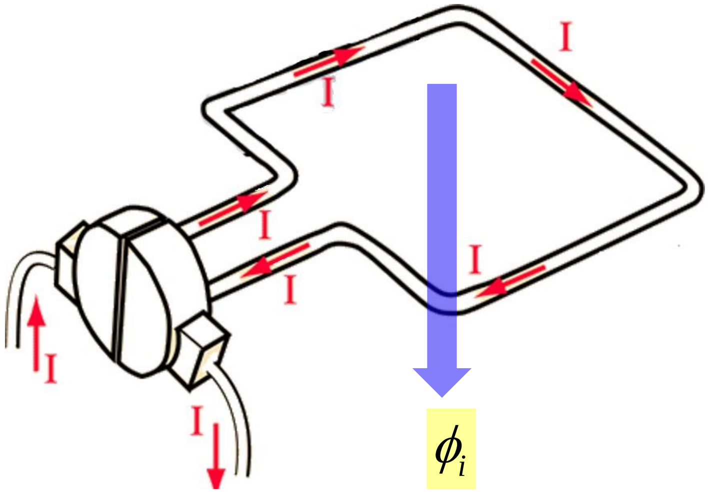


兩個磁場產生
的吸引力

永久磁石
產生之磁場

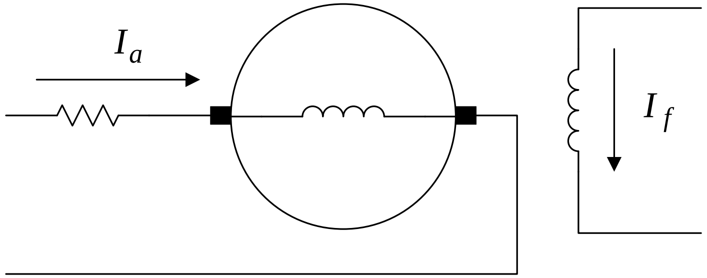


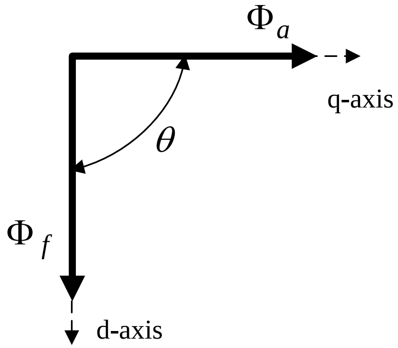
電流產生磁場
Ampere’s law



控制架構 – 永磁同步馬達控制原理

□直流馬達輸出轉矩產生方式 -磁場交互作用





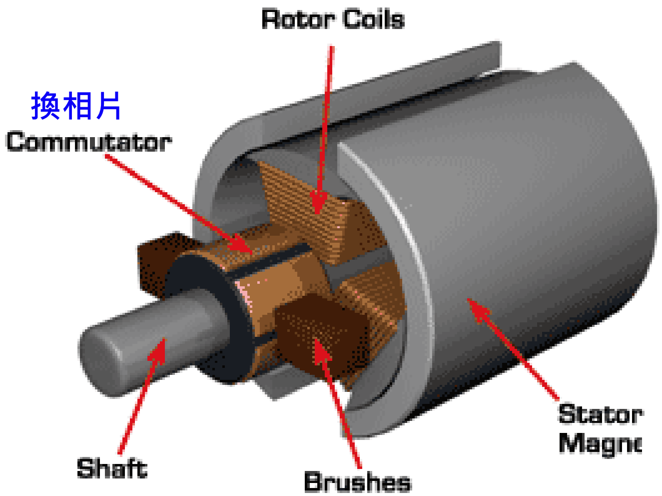


Figure 1 brushed DC motor↵

https://dynamictestingonsteppermotor.wordpress.com/2015/03/01/brushed-dc-motor/

https://www.pinterest.com/pin/3025924732537737/

$$T_e \propto \Phi_a \Phi_f \sin(\theta)$$

$$\text{if} \quad \Phi_a \propto I_a, \Phi_f \propto I_f \text{ and } I_f = \text{constant}$$

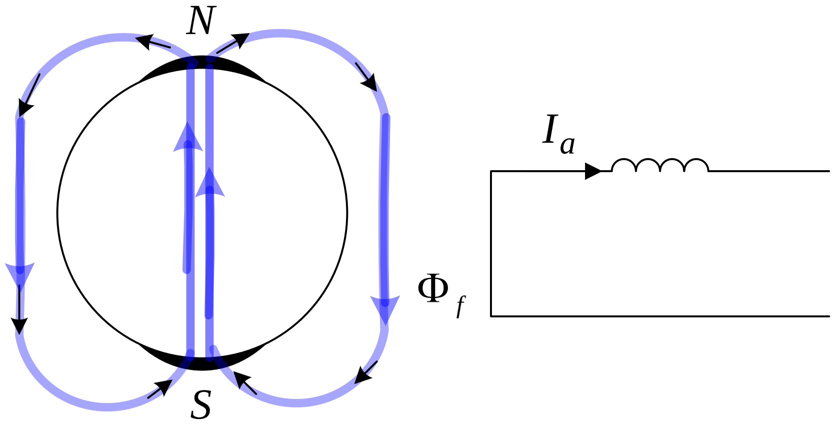
$$T_e \propto \Phi_a \sin(\theta)$$

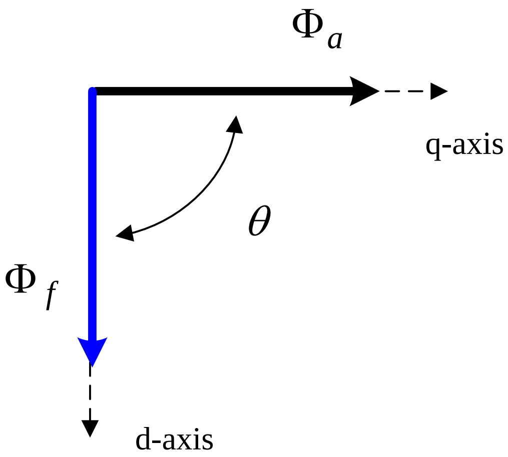
$$\text{if} \quad \theta = 90^\circ$$ (由換相片及碳刷完成)

$$T_e \propto \Phi_a \Rightarrow T_e \propto I_a$$

控制架構 – 馬達產生輸出轉矩之原理

□永磁同步馬達輸出轉矩產生方式 - 磁場交互作用





$$T_e \propto \Phi_a \, \Phi_f \, \sin(\theta)$$

$$\text{if} \quad \Phi_a \propto I_a$$

$$T_e \propto \Phi_a \, \sin(\theta)$$

$$\text{if} \quad \theta = 90^\circ$$ 電子換相來達成

- 加入感測轉子磁場位置之感測元件
- 以電壓及(或)電流感測元件之訊號來估算轉子磁場位置 (Position sensorless)

$$T_e \propto \Phi_a \Rightarrow T_e \propto I_a$$

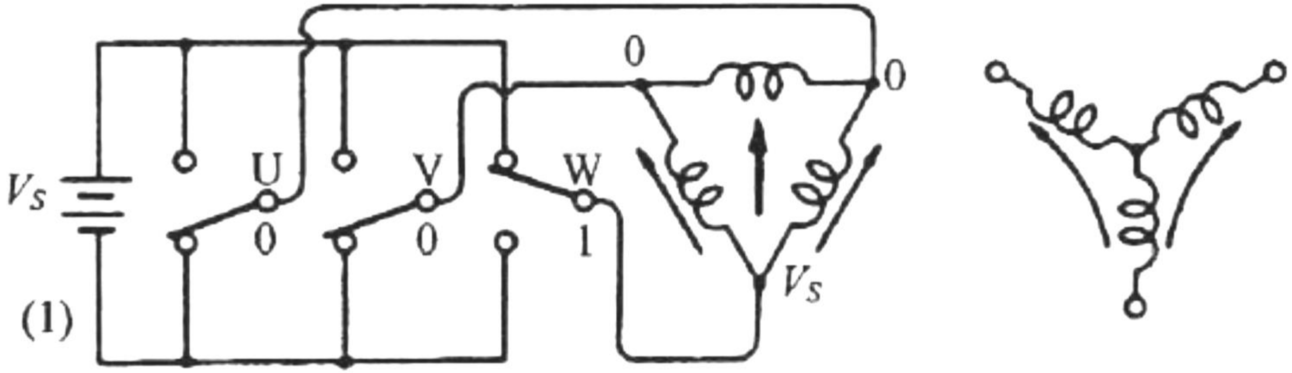
如何由輸入定子線圈之三相電流產生旋轉磁場？

控制架構 – 馬達產生輸出轉矩之原理

□定子線圈產生旋轉磁場 – 電壓驅動產生電流，再由電流產生磁場

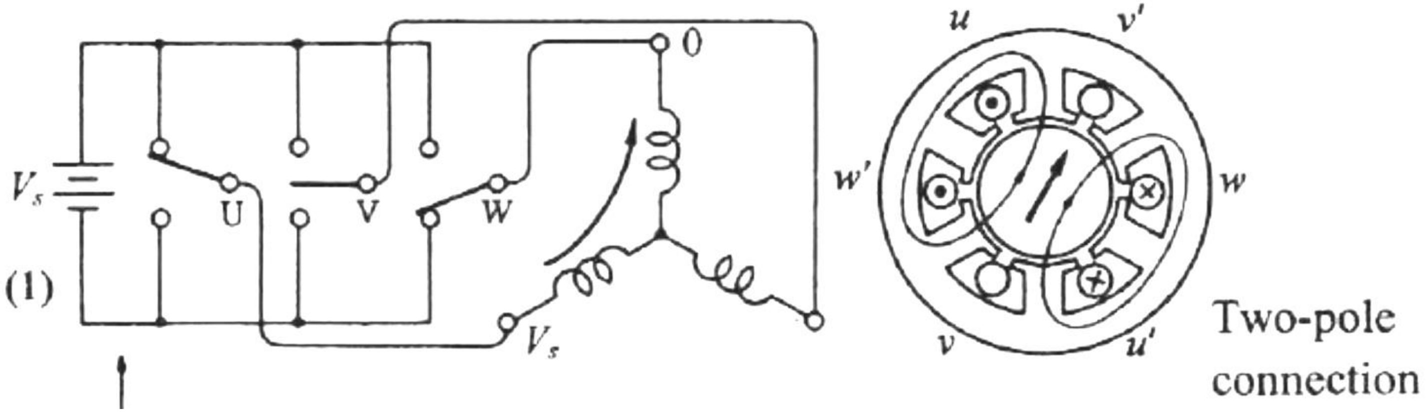
假設三相線圈為Y接且為平衡，亦即線圈相阻抗相同及相反電動勢為平衡。

180° commutation



目前電壓驅動之主流

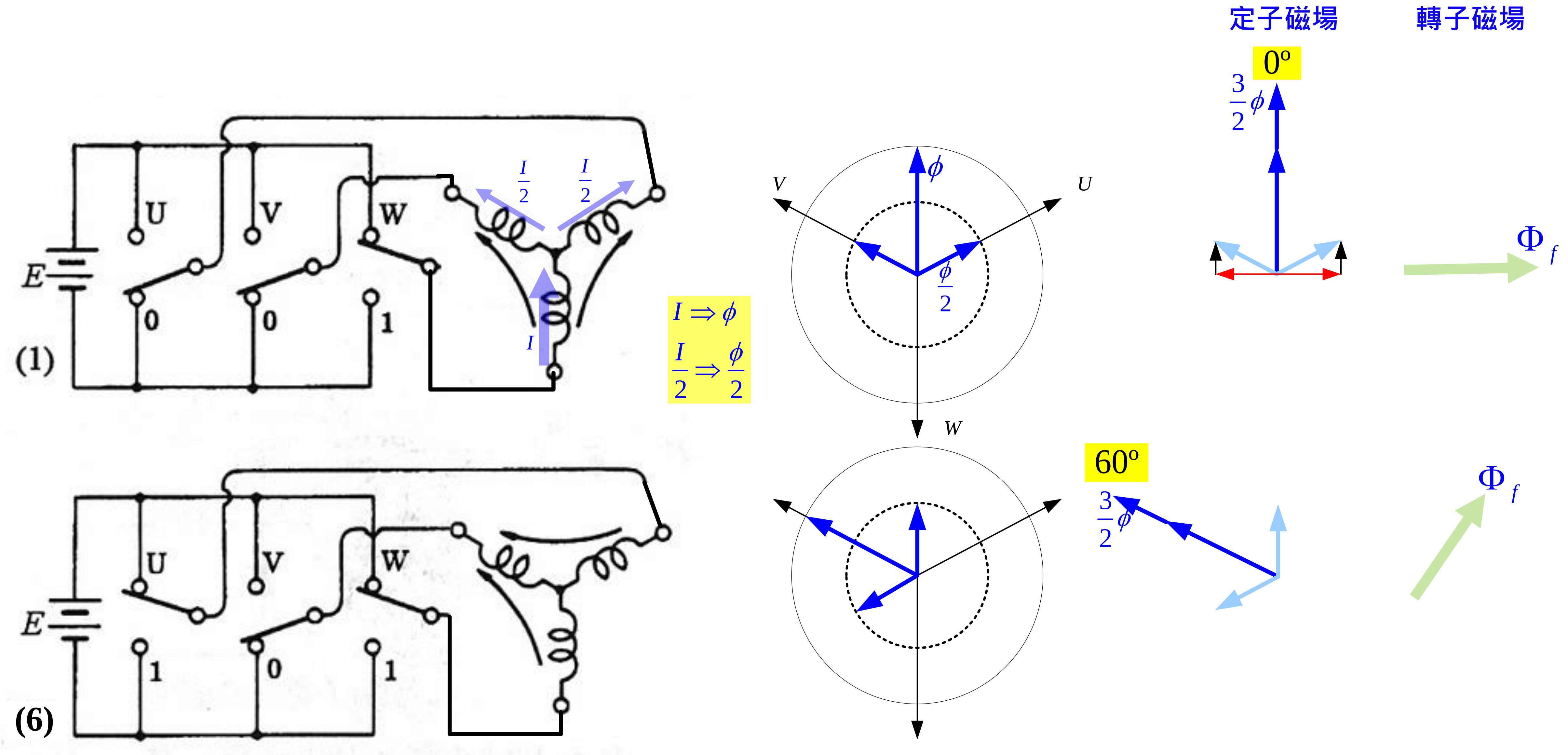
120° commutation



Ref: Takashi Kenjo, "Brushless motors, Advanced Theory and Modern Applications", 2003

控制架構 – 馬達產生輸出轉矩之原理

□定子線圈產生旋轉磁場 –電壓驅動產生電流，再由電流產生磁場



$I \Rightarrow \phi$

$\frac{I}{2} \Rightarrow \frac{\phi}{2}$

Ref: Takashi Kenjo, "Brushless motors, Advanced Theory and Modern Applications", 2003

控制架構 – 馬達產生輸出轉矩之原理

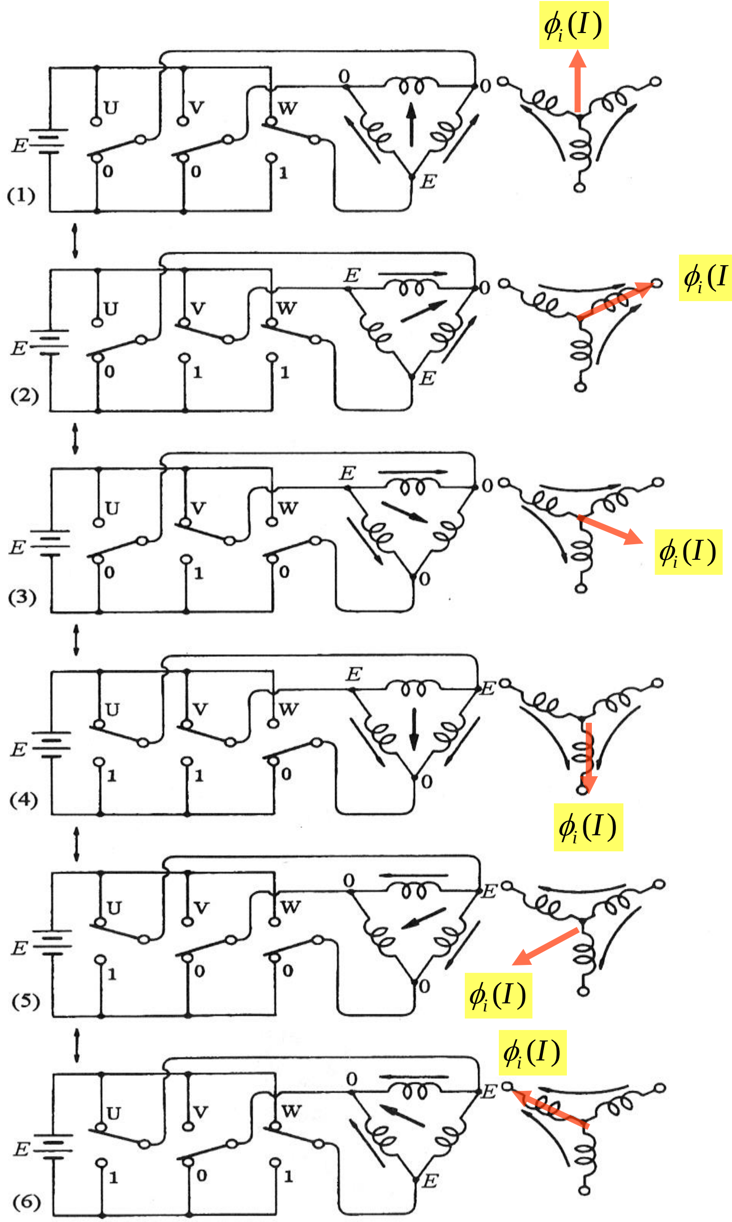
□定子線圈產生旋轉磁場 –電壓驅動產生電流，再由電流產生磁場

- ✓ 由不同的開關組合依序產生旋轉的磁場，目前說明為兩極之三相線圈及粗略使用開關阻態，因此只區分為六段；
- ✓ 使用正弦脈寬調變(SPWM)可讓旋轉磁場解析度提高；
- ✓ 輸入線圈電流大小可經由脈寬調變來間接調控。

$$T = K\,\phi_i(I)\,\phi$$

$$T = k_t I$$

調控輸入線圈電流 >> 調控輸出轉矩



Ref: Takashi Kenjo, “Brushless motors, Advanced Theory and Modern Applications”, 2003

控制架構 – 同步框電流控制

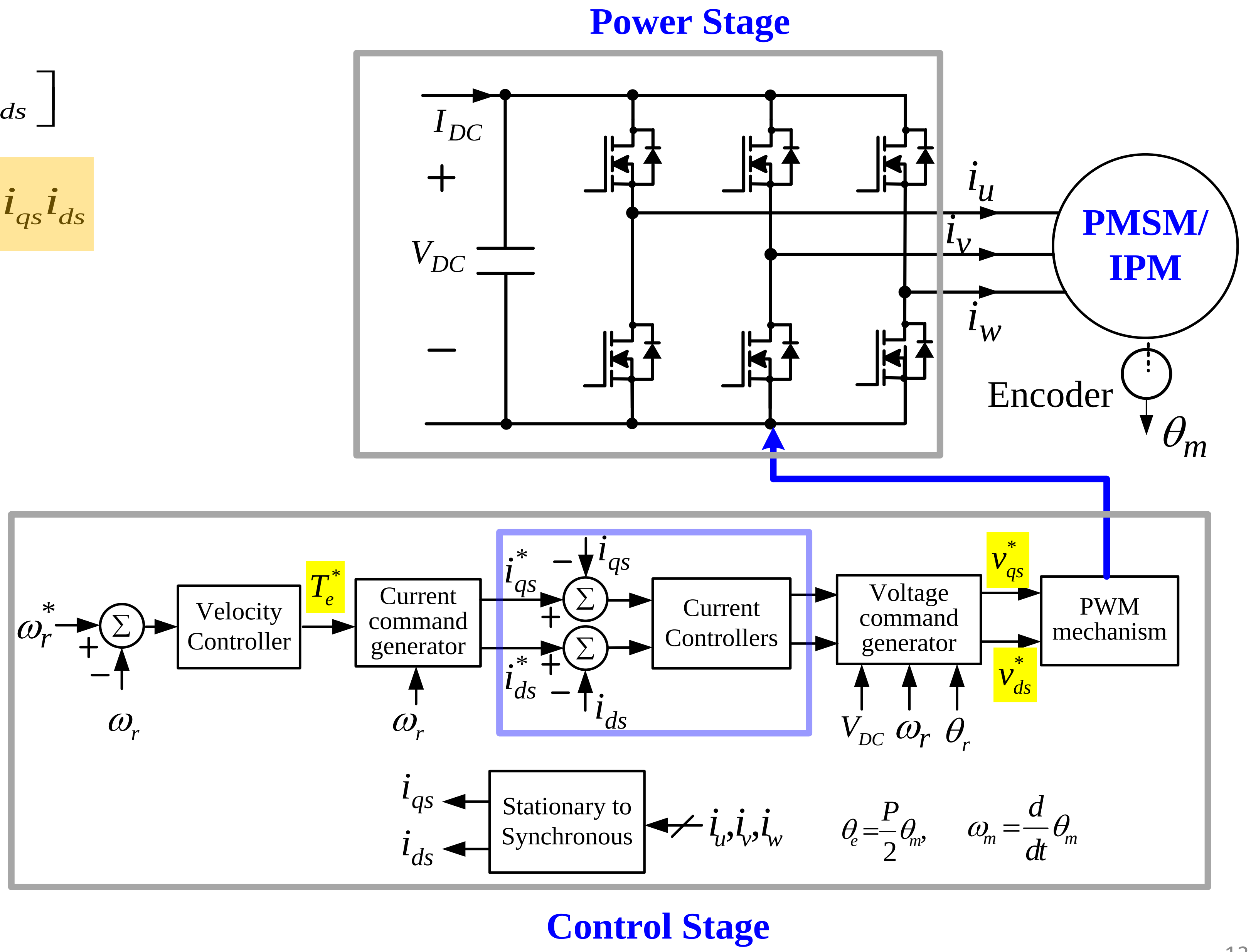
輸出轉矩方程式

$$T_e = \frac{3}{2}\frac{P}{2}\left[\lambda_m i_{qs} + (L_{ds} - L_{qs}) i_{qs} i_{ds}\right]$$

$$= \underbrace{\frac{3}{2}\frac{P}{2}\lambda_m i_{qs}}_{\text{電磁轉矩}} + \underbrace{\frac{3}{2}\frac{P}{2}(L_{ds} - L_{qs}) i_{qs} i_{ds}}_{\text{磁阻轉矩}}$$

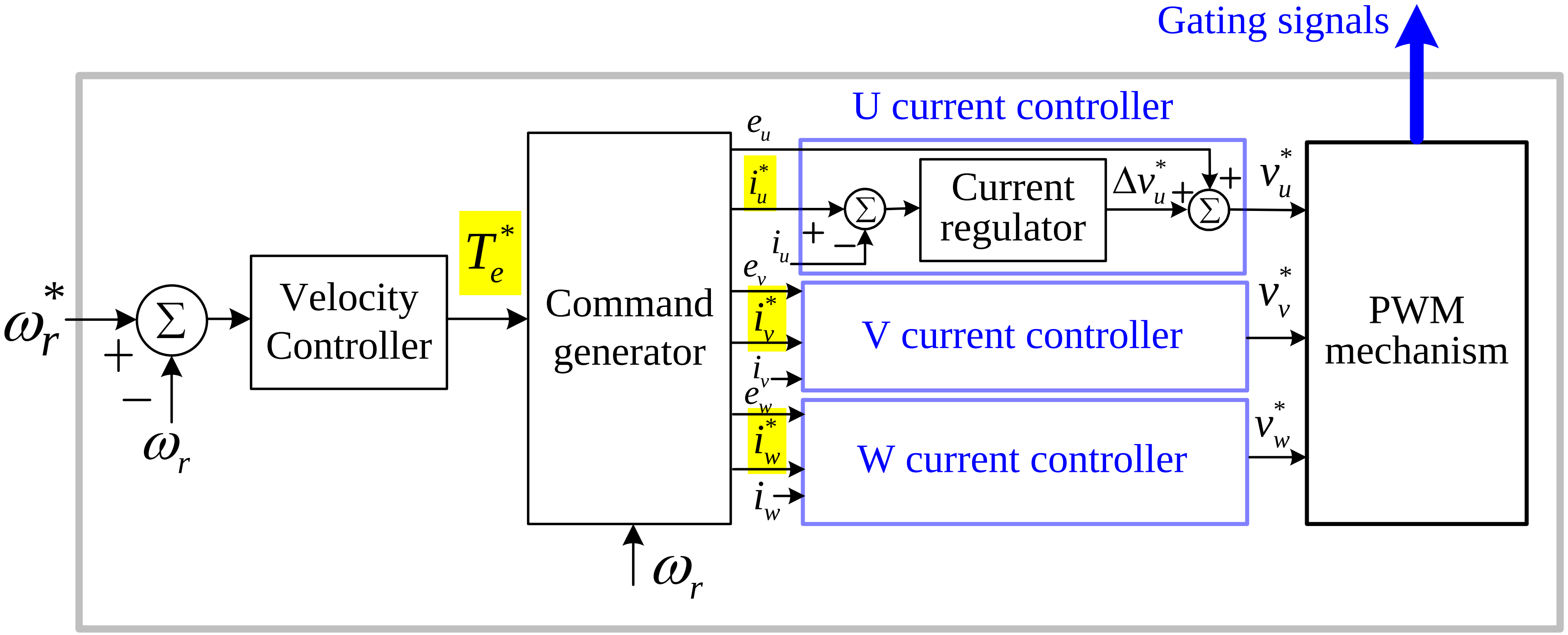
$$= f(i_{qs})$$

$$T_e^* \Rightarrow \begin{bmatrix} i_{qs}^* \\ i_{ds}^* \end{bmatrix} \Rightarrow \begin{bmatrix} v_{qs}^* \\ v_{ds}^* \end{bmatrix}$$



控制架構 – 靜止框三相獨立電流控制

$$T_e^* \Rightarrow \begin{bmatrix} i_u^* \\ i_v^* \\ i_w^* \end{bmatrix} \Rightarrow \begin{bmatrix} v_u^* \\ v_v^* \\ v_w^* \end{bmatrix}$$



Control stage

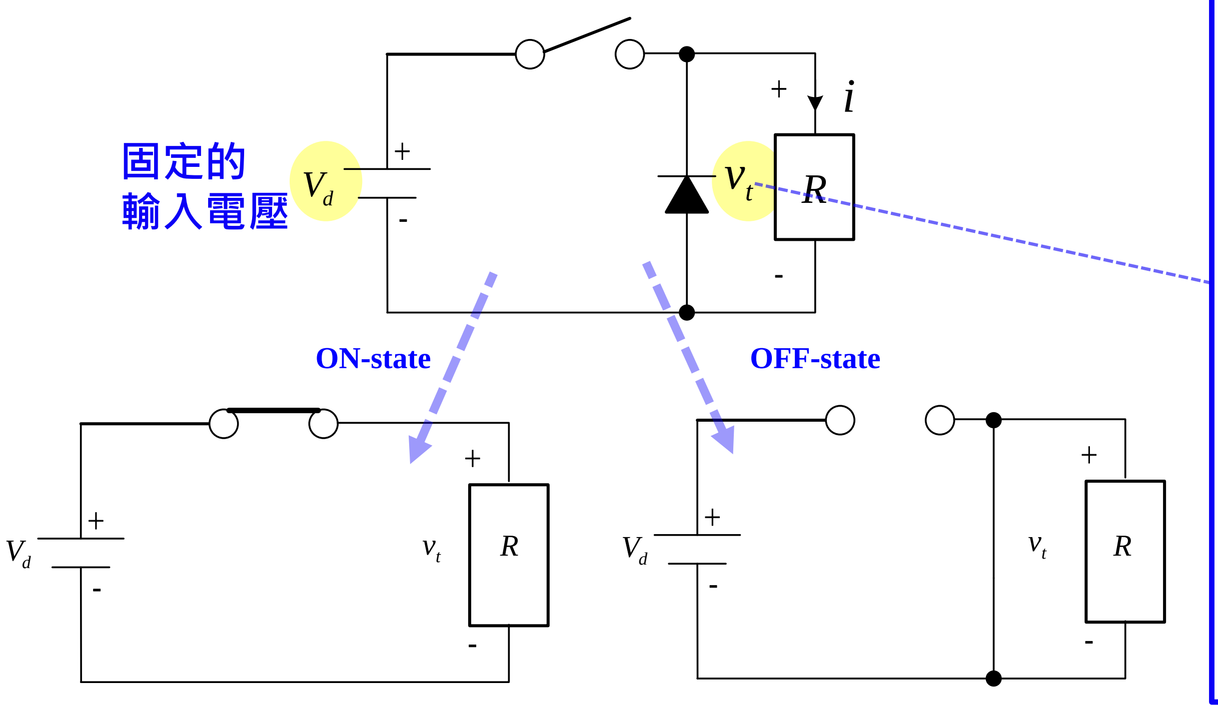
控制架構 – 脈寬調變

□ 脈波寬度調變技術(PWM, Pulse Width Modulation)

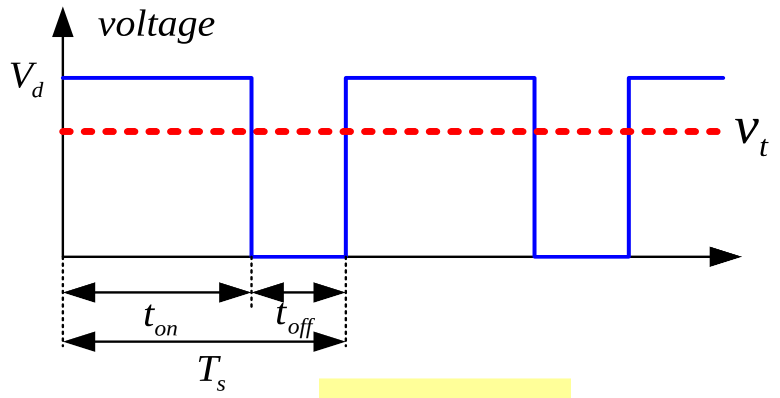
- ✓ 取代過去以電晶體進行功率線性放大>>效率差
- ✓ 將功率晶體操作在非線性區>>以On-Off取代線性放大，可有效地降低功率晶體損失

□ PWM目的是在固定輸入電壓 V_d 下，調控輸出電壓 v_t 達到負載電流 i 控制的目的。

$$v_t = DV_d \Rightarrow i = \frac{v_t}{R}$$



輸出電壓



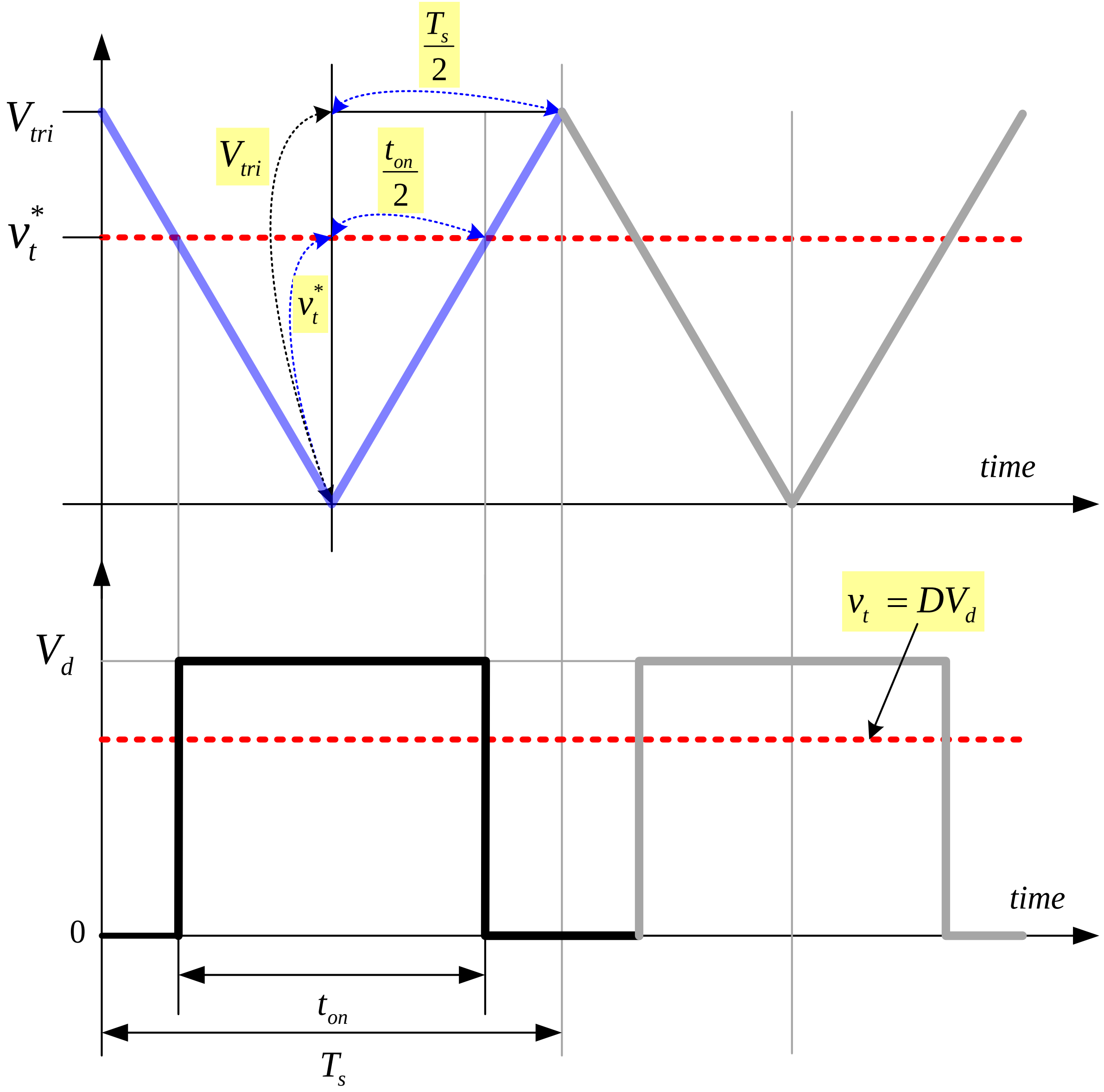
$v_t = DV_d$ 類似線性放大

$$v_t = V_{ave} = \frac{1}{T_s}\int_0^{T_s} v(t)dt = \frac{1}{T_s}[V_d t_{on} + 0 \times t_{off}] = \frac{t_{on}}{T_s}V_d = DV_d$$

$$D \equiv \frac{t_{on}}{T_s} : duty, T_s : switching\ period$$

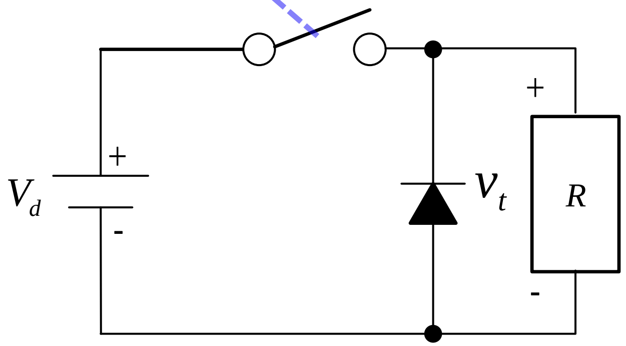
控制架構 – 脈寬調變

□ PWM實現方法



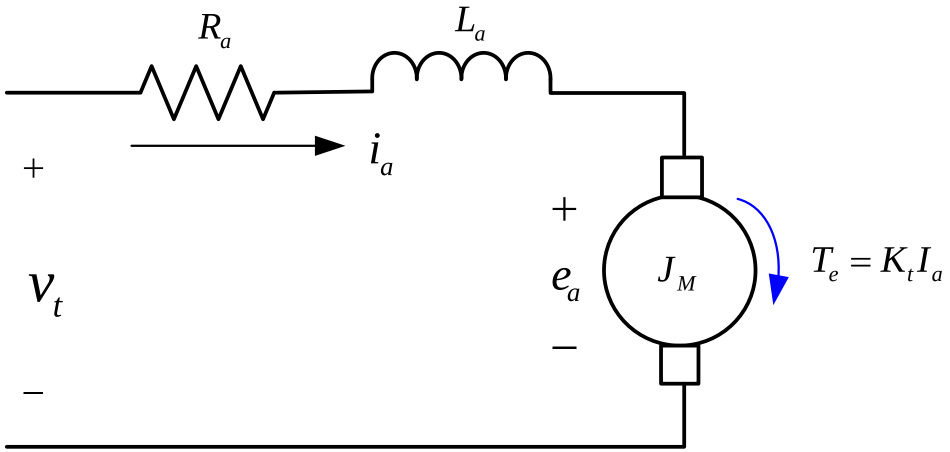
$$\frac{V_{tri}}{\frac{T_s}{2}} = \frac{v_t^*}{\frac{t_{on}}{2}} \Rightarrow \frac{v_t^*}{V_{tri}} = \frac{t_{on}}{T_s}$$

$$t_{on} = \frac{v_t^*}{V_{tri}} T_s = DT_s$$



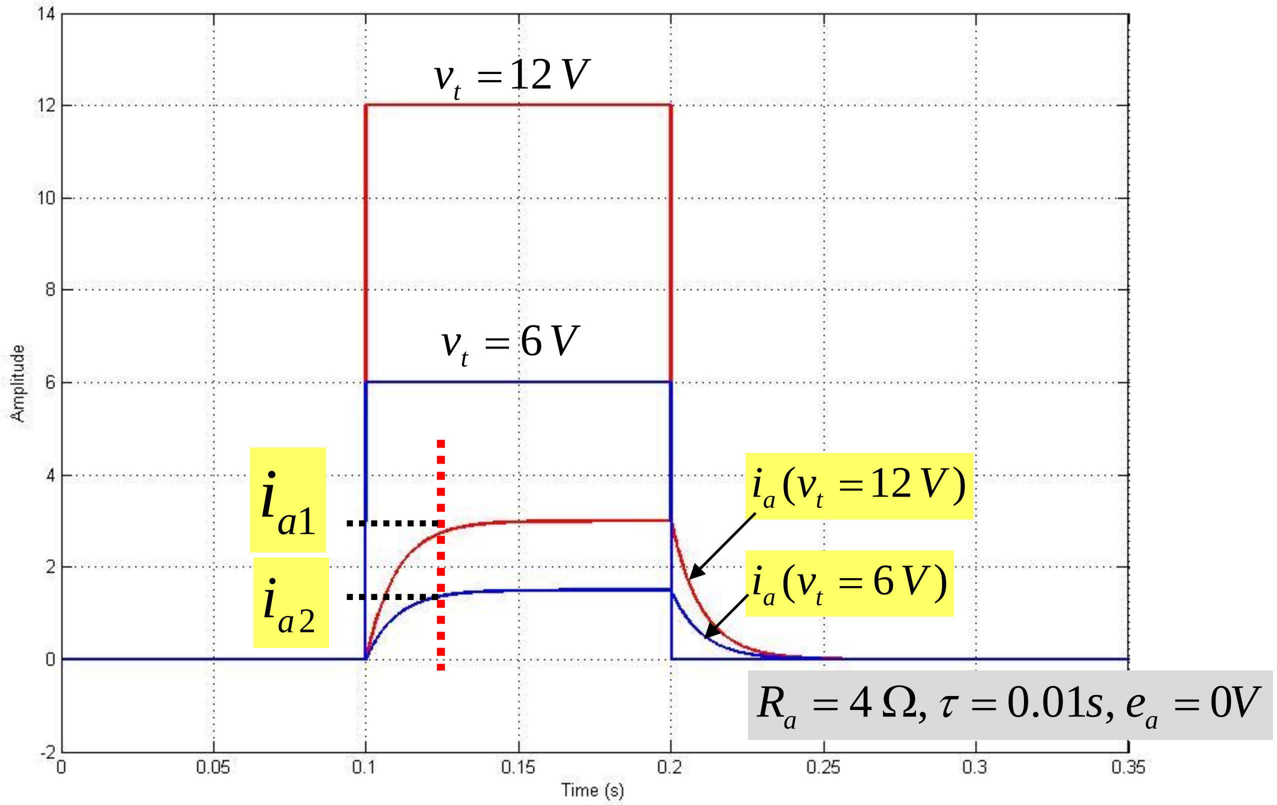
控制架構 – 脈寬調變

□ 馬達電氣時間常數及電壓差



$$i_a(t) = \frac{(v_t - e_a)}{R_a}(1 - e^{-\frac{t}{\tau}})$$

$\tau = \frac{L_a}{R_a}$:時間常數
time constant



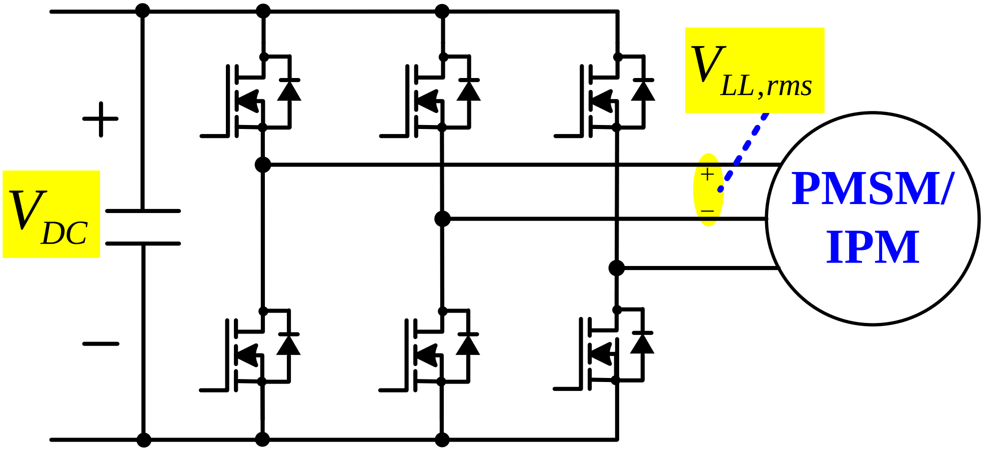
相同電氣參數下，電壓差 $(v_t - e_a)$ 愈大，在相同時間下電流愈大，亦即輸出轉矩響應較快

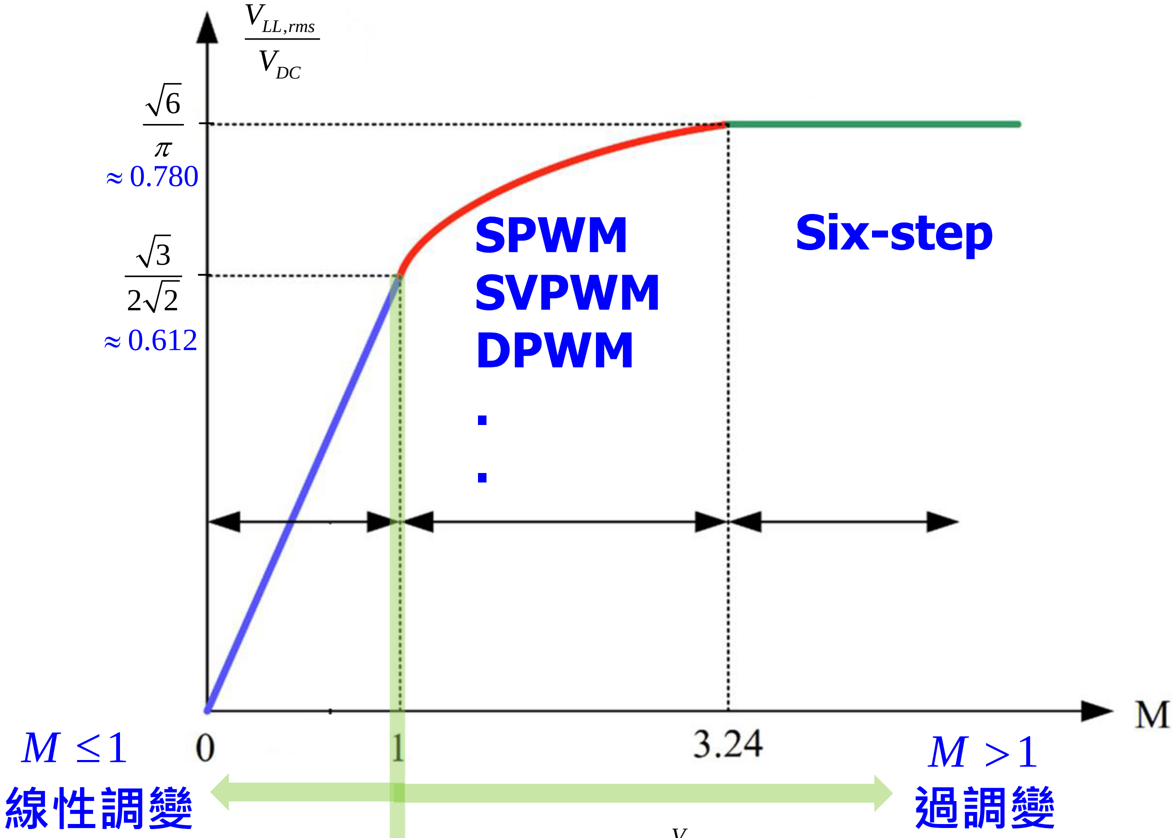
$$i_{a1} \geq i_{a2} \Rightarrow T_{e1}(= K_t i_{a1}) \geq T_{e2}(= K_t i_{a2})$$

馬達在高轉速區可使用過調變來提高輸出轉矩響應

控制架構 – 脈寬調變

- □ 過調變 – 針對正弦波的**相電壓命令** $v^*(t)$，目的是在有限直流鏈電壓 V_{DC} 下提高輸出有效線電壓 $V_{LL,rms}$



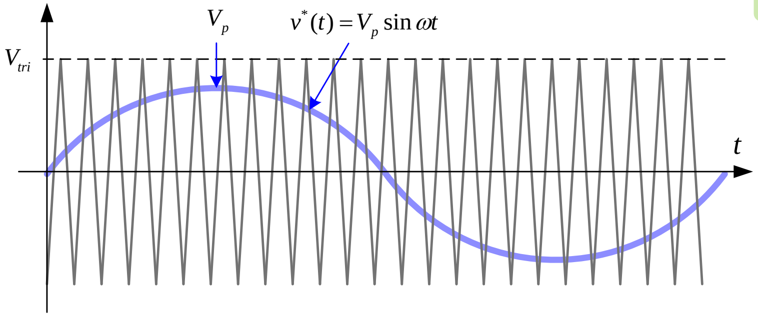


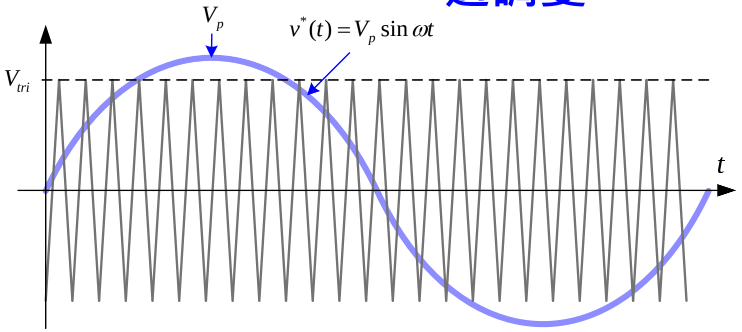
調變指數
Modulation index

$$M \triangleq \frac{V_p}{V_{tri}}$$

$$v^*(t) = V_p \sin \omega t$$

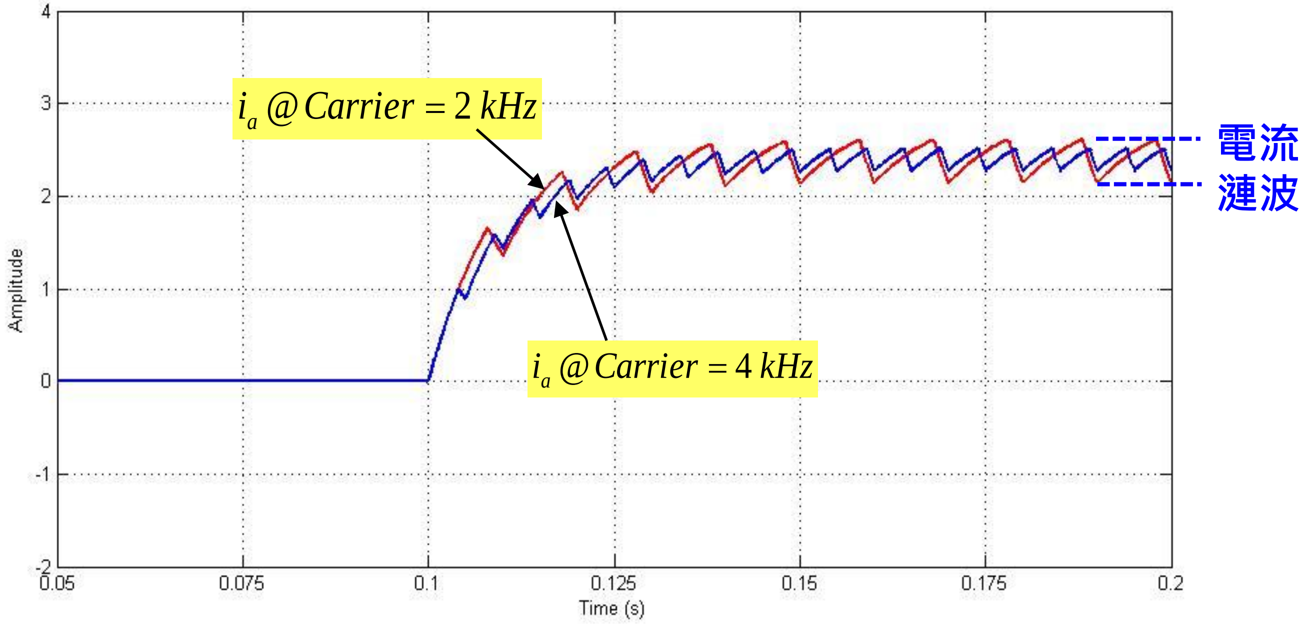
V_{tri} 三角波峰值





□ 電流漣波、切換頻率及馬達損失

- ✓ 較大電流漣波會增加馬達損失(銅損+鐵損)
- ✓ 提高功率晶體切換頻率可有效降低電流漣波
- ✓ 一般IGBT或Si MOSFET提高切換頻率會增加切換損失；
- ✓ SiC MOSFET具較低的切換損失，因此可提高切換頻率來降低馬達損失及噪音。



SiC MOSFET with asynchronous PWM

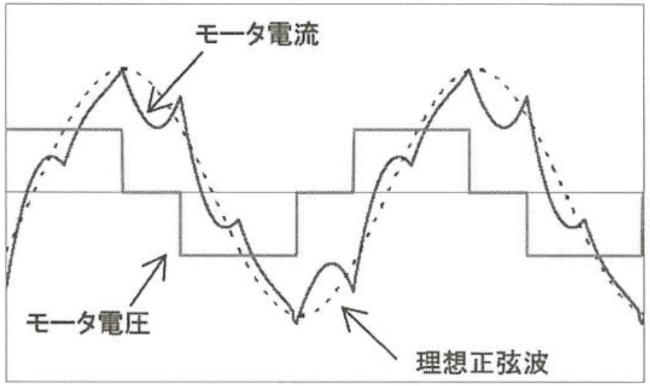


図20. 1パルス駆動時電圧・電流波形

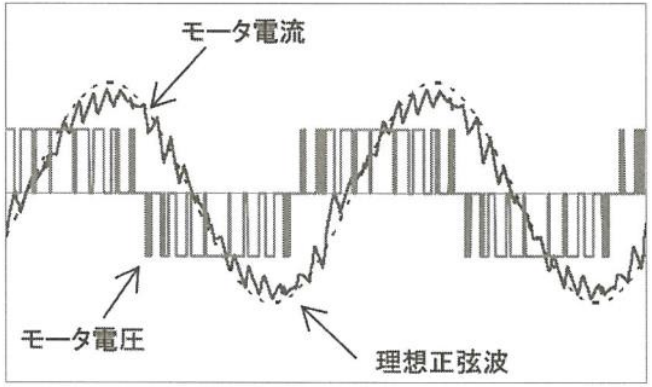


図21. 非同期PWM・多パルス駆動時電圧・電流波形

- ・SiCの低損失特性を活かしたスイッチング周波数向上によりモータの高周波損失を削減
- ・V/f 傾きを下げたモータ設計により非同期PWM・多パルス運転範囲を拡大　モータ低損失化、低騒音化に寄与
  （最大40%減）　（最大7dB減）

Ref: 37th Motor Tech. Symposium, Techno-Frontier 2017

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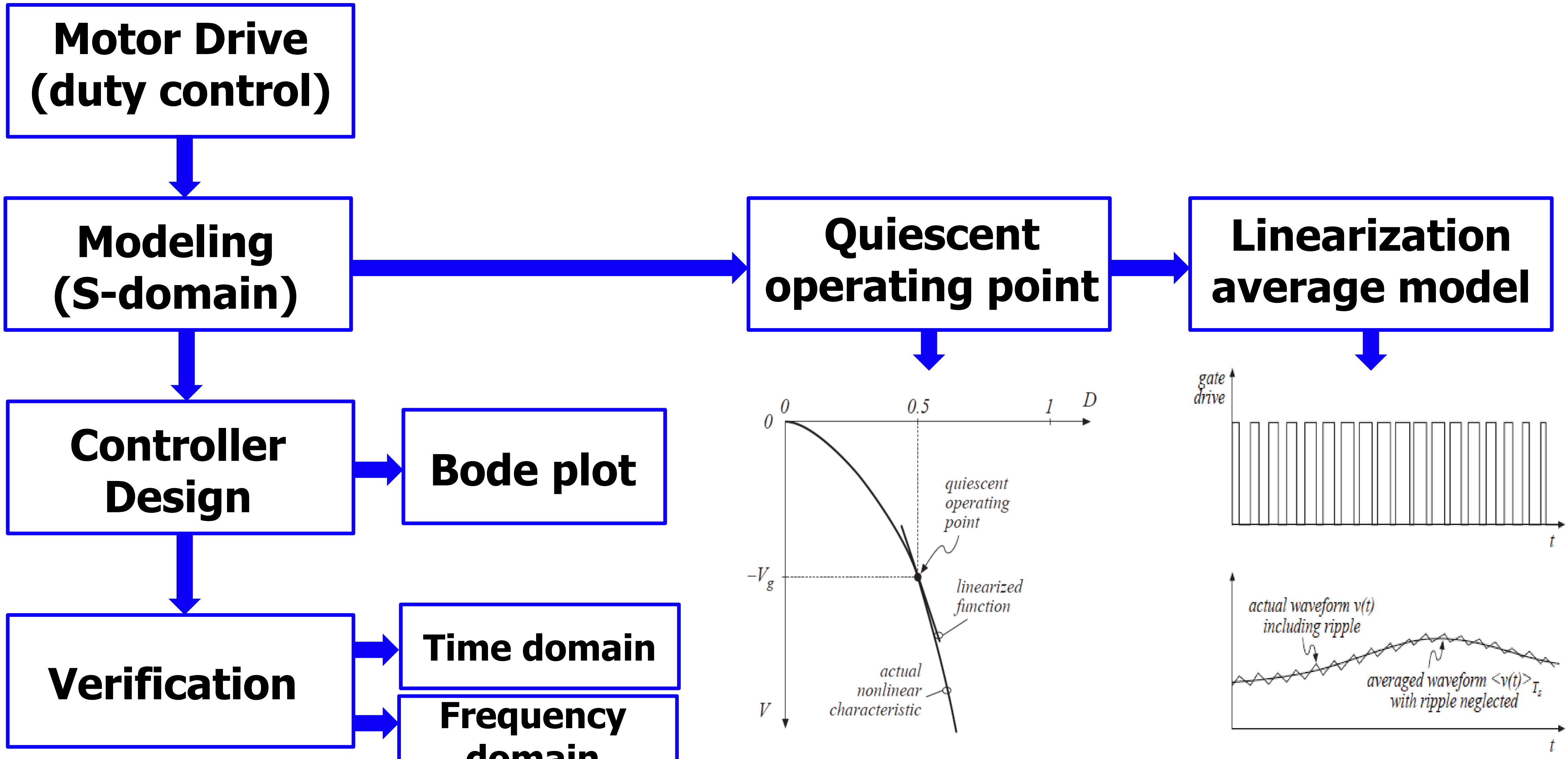
電流迴路設計 – 設計程序



Motor Drive (duty control)

Modeling (S-domain)

Controller Design

Bode plot

Verification

Time domain

Frequency domain

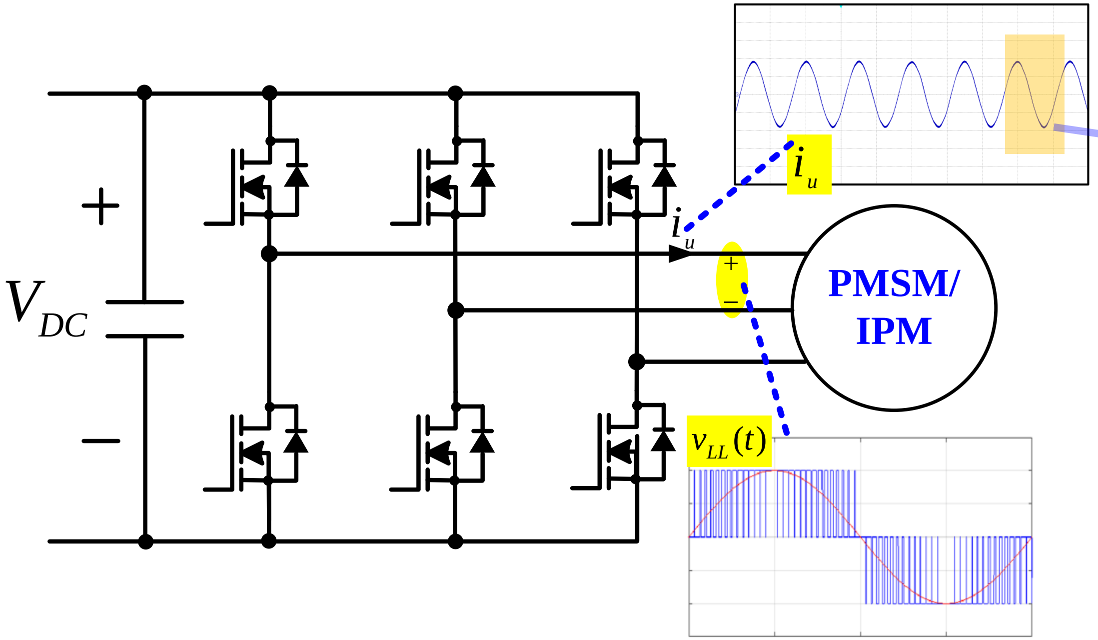
Quiescent operating point

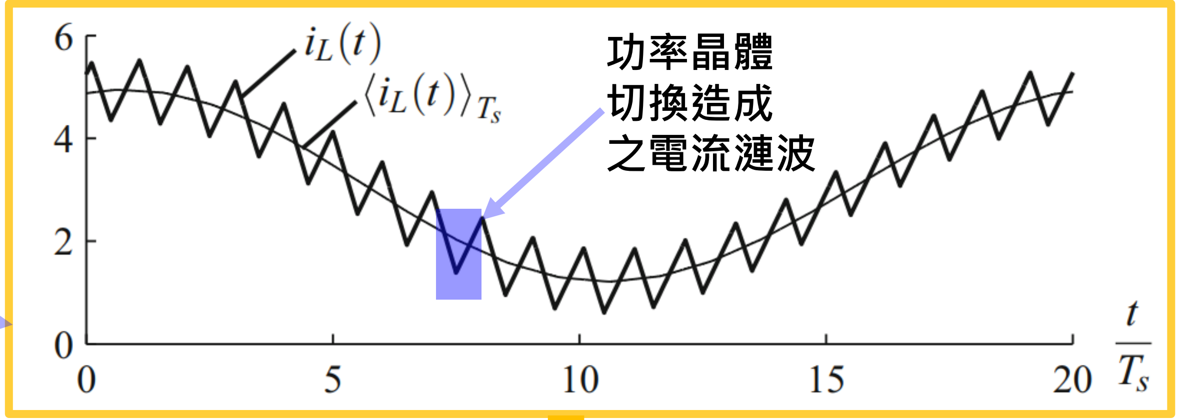
Linearization average model

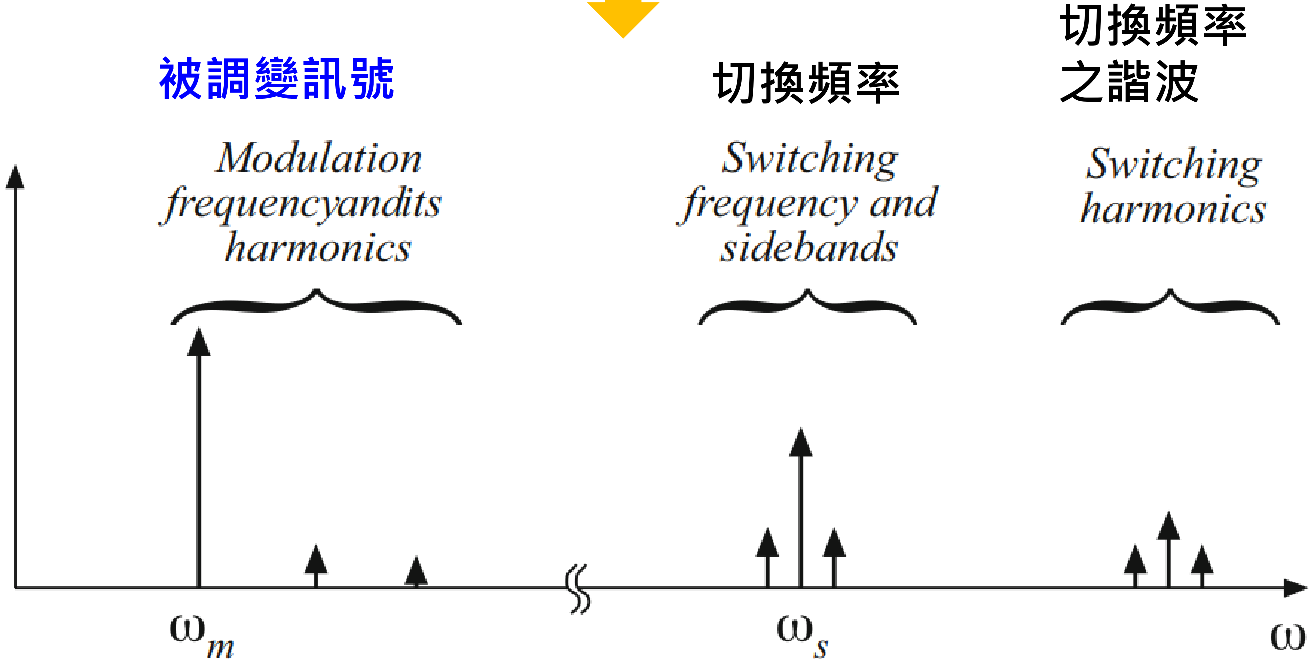
Ref. : Erickson, "Fundamentals of Power Electronics" 3rd

電流迴路設計 – 頻譜

□ 電流迴路之頻率範圍







Ref.：Erickson, “Fundamentals of Power Electronics” 3rd

電流迴路設計 – 頻譜

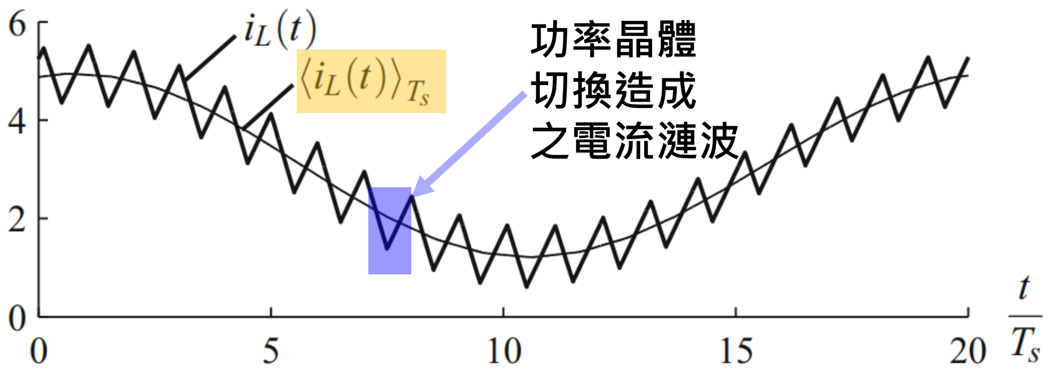
□ 電流漣波去除方法

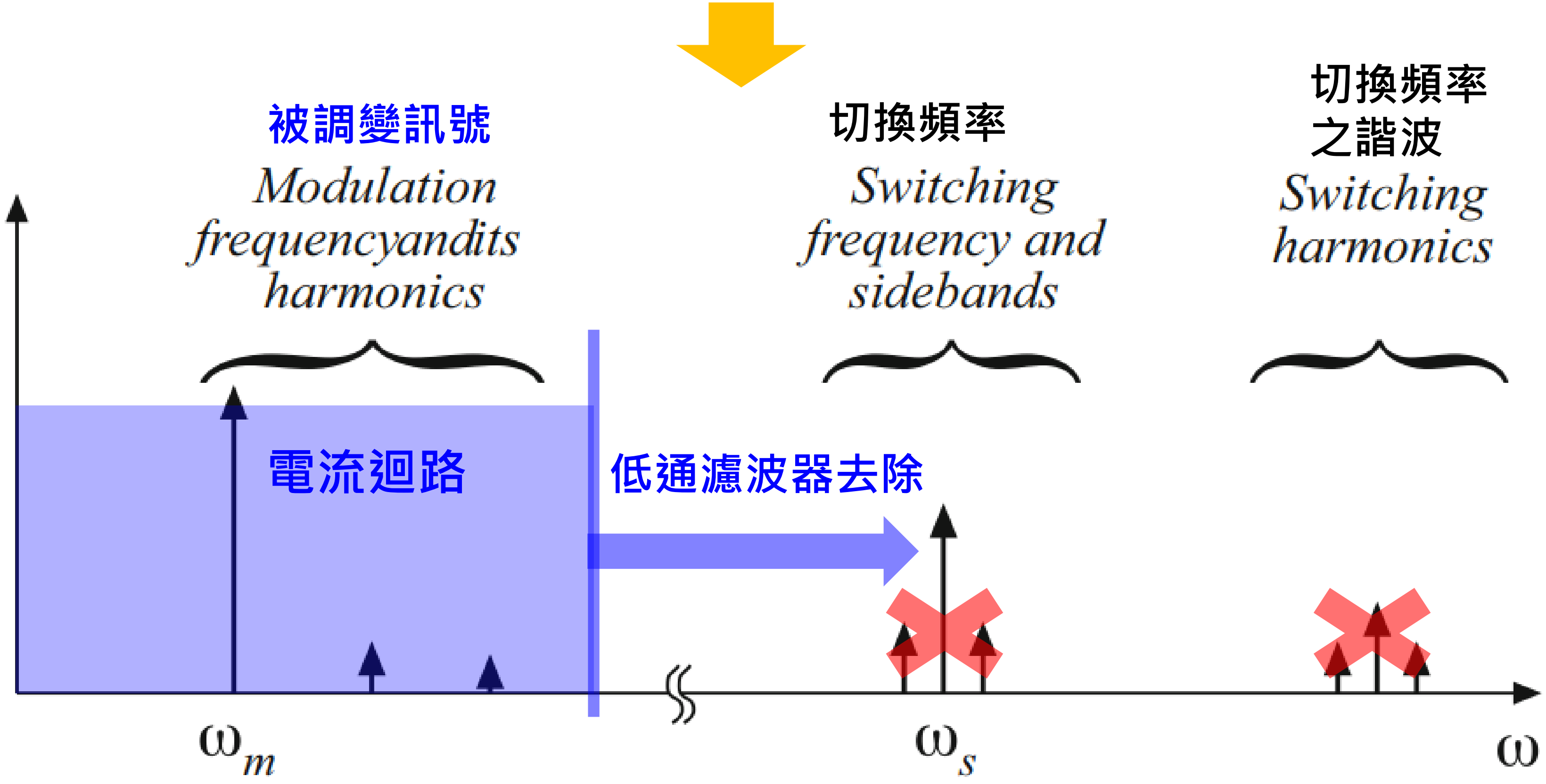
✓切換週期平均值 > 理論分析

$$\langle x(t)\rangle_{T_s} = \frac{1}{T_s}\int_{t-T_s/2}^{t+T_s/2} x(\tau)d\tau$$

T_s :切換週期, $\omega_s = \dfrac{2\pi}{T_s}$

✓低通濾波器 > 實現系統 (SW+HW)





電流迴路設計 – Bode Plot

□ Bode plot 特點

- ✓ 適用於線性系統
- ✓ 穩定度分析
- ✓ 動態響應

Phase margin φ_m
Crossover frequency ω_c

□ Second order system

Frequency domain

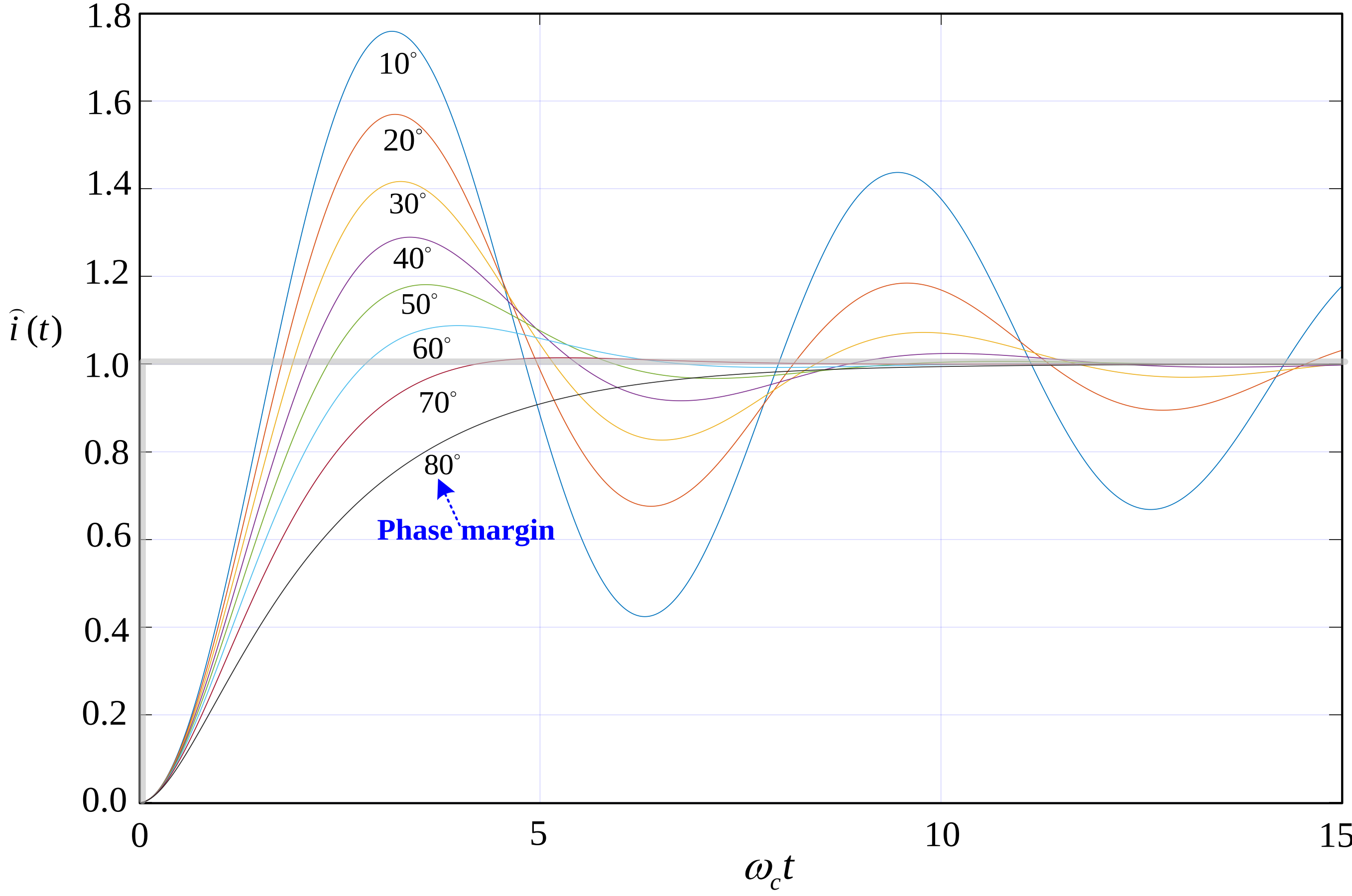
$$\frac{T(s)}{1+T(s)}=\frac{1}{1+\frac{s}{Q\omega_c}+(\frac{s}{\omega_c})^2}$$

$$\Updownarrow$$

Time domain
(Unit step response)

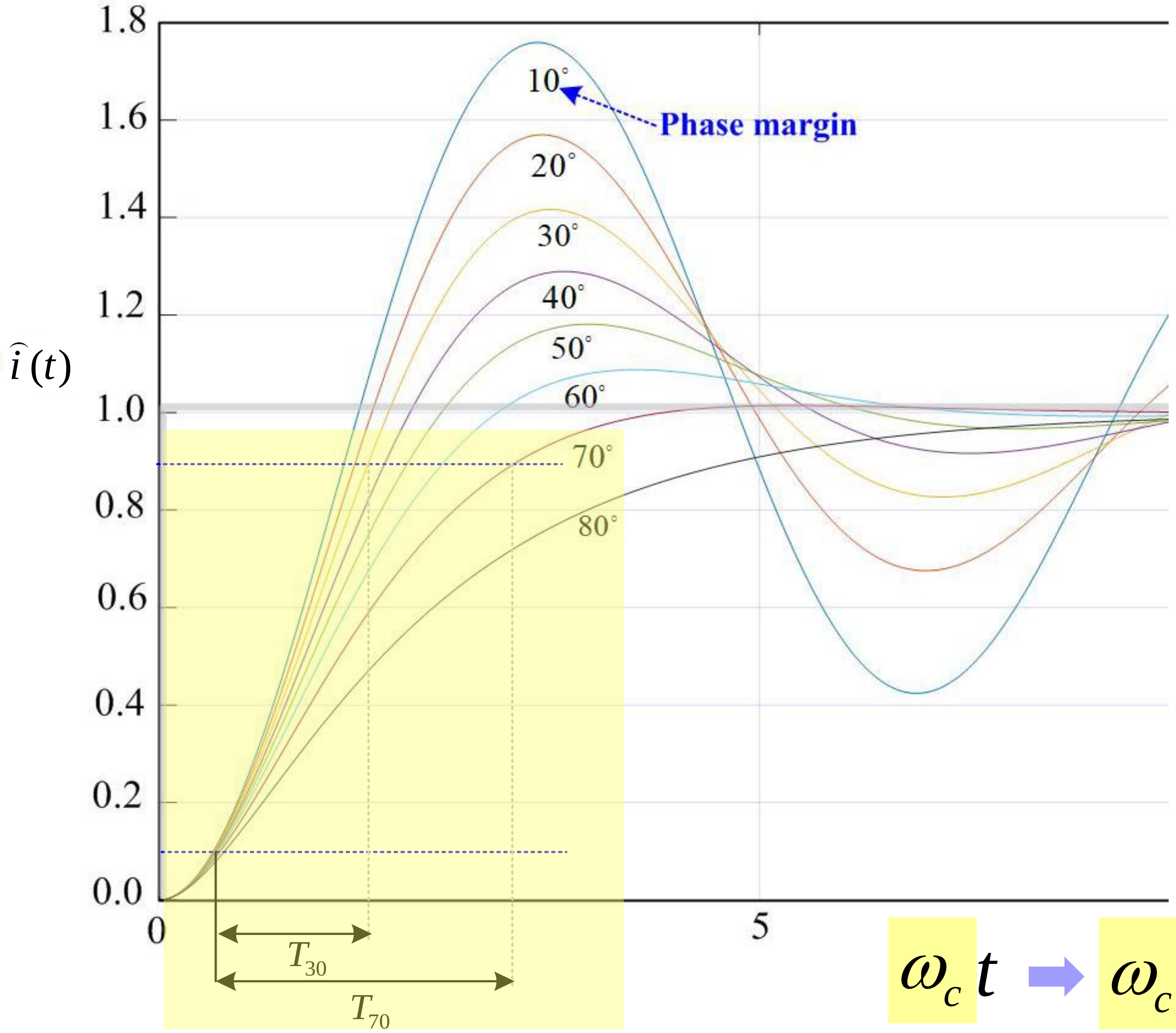
$$\hat{i}(t)=1-\frac{2Qe^{-\omega_c t/2Q}}{\sqrt{4Q^2-1}}\sin\left[\frac{\sqrt{4Q^2-1}}{2Q}\omega_c t+\tan^{-1}(\sqrt{4Q^2-1})\right]$$

$$Q=\frac{\sqrt{\cos(\varphi_m)}}{\sin(\varphi_m)}$$



電流迴路設計 – Bode Plot

□ 單位步階響應的上升時間 (2nd order system)



$\omega_c t \Rightarrow \omega_c \uparrow t \downarrow$

ω_c 愈大及較小的 φ_m 皆會縮短rising time

T_{30} : **Rising time @** $\varphi_m = 30^\circ$

T_{70} : **Rising time @** $\varphi_m = 70^\circ$

電流迴路設計 – Bode Plot

□ 電流迴路之小訊號模型 (假設電氣參數及負載轉矩不變)

工作點　擾動量

$$v_{uN}(t) = V_{uN0} + \Delta v_u(t)$$

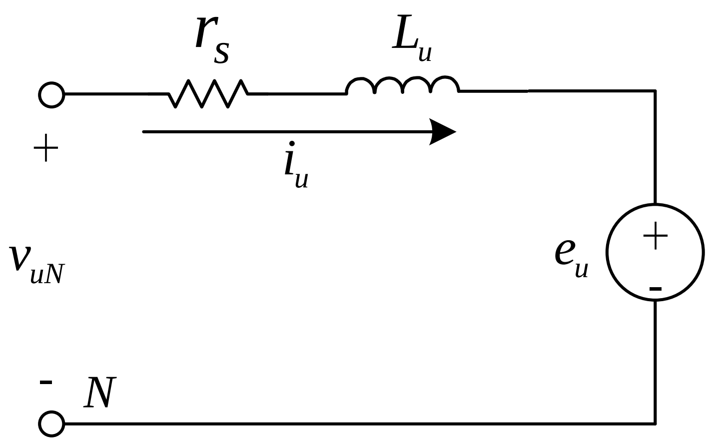
$$i_u(t) = I_{u0} + \Delta i_u(t)$$

$$e_u(t) = E_{uN0} + \Delta e_u(t)$$

$$T(t) = T_0 + \Delta T(t)$$

$$\omega(t) = \omega_0 + \Delta\omega(t)$$

輸入電壓變動造成其他訊號的擾動



$$v_{uN} = L_u \frac{d}{dt} i_u + r_s i_u + e_u$$

$$V_{uN0} + \Delta v_u = (L_u \frac{d}{dt} I_{u0} + r_s I_{u0} + E_{uN0}) + L_u \frac{d}{dt}\Delta i_u + r_s \Delta i_u + \Delta e_u$$

去除工作點

$$V_{uN0} = L_u \frac{d}{dt} I_{u0} + r_s I_{u0} + E_{uN0}$$

小訊號方程式 $\Delta v_u = L_u \frac{d}{dt}\Delta i_u + r_s \Delta i_u + \cancelto{0}{\Delta e_u}$

Laplace transform

小訊號模型 $\dfrac{\Delta i_u(s)}{\Delta v_u(s)} = \dfrac{1}{L_u s + r_s}$

機械功=電氣功

$$\omega T = 3 e_u i_u = 3 K_e \omega i_u$$

$$\begin{cases} T = 3K_e i_u \\ T = J\frac{d}{dt}\omega + B\omega + T_L \end{cases}$$

$$\Delta T = 3K_e \Delta i_u$$

$$\Delta T = J\frac{d}{dt}\Delta\omega + B\Delta\omega$$

$$\Delta e_u = K_e \Delta\omega$$

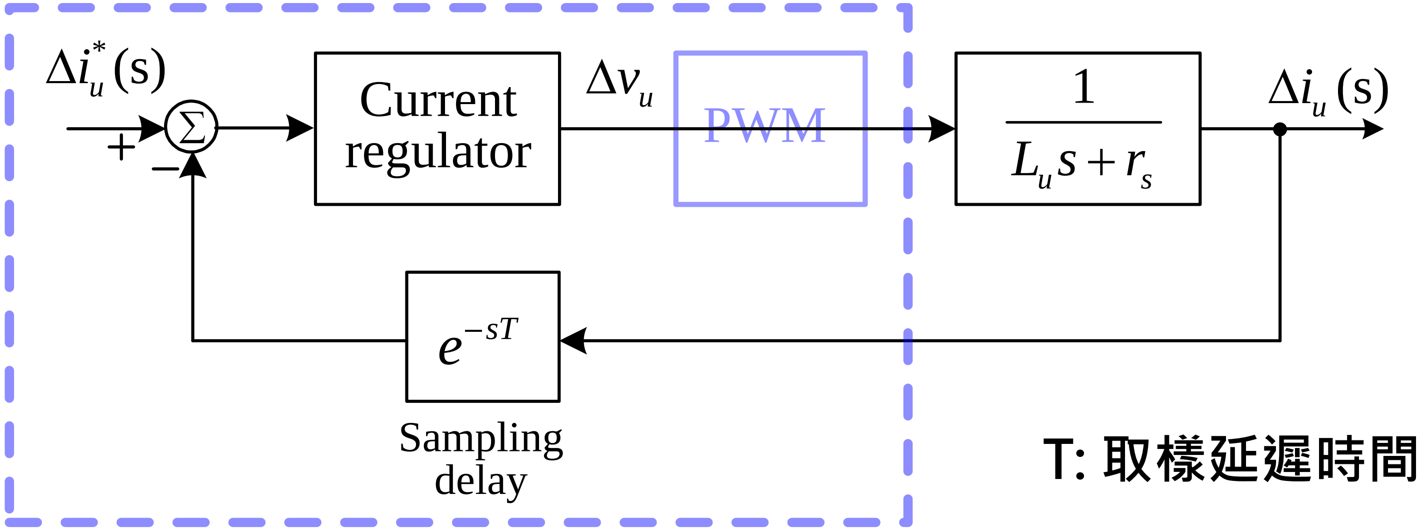
$\Delta i_u \downarrow \quad J \uparrow$

$$\Delta e_u = K_e \Delta\omega \approx 0$$

馬達(含負載)慣量大及擾動電流小，可忽略反電動勢的變動

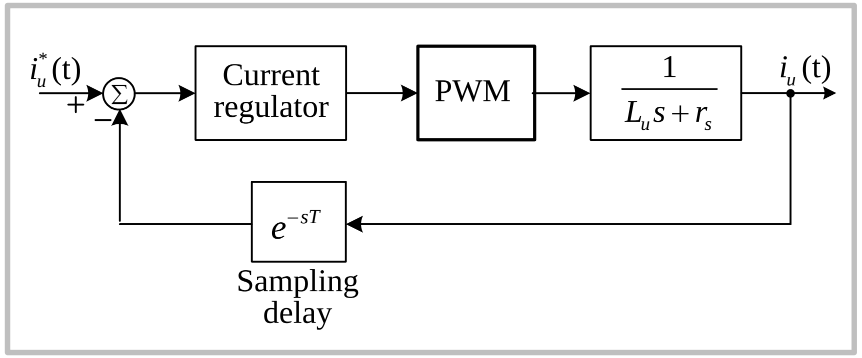
電流迴路設計 – Bode Plot

□ 電流迴路之小訊號控制架構



T: 取樣延遲時間

□ 電流迴路控制架構



大綱

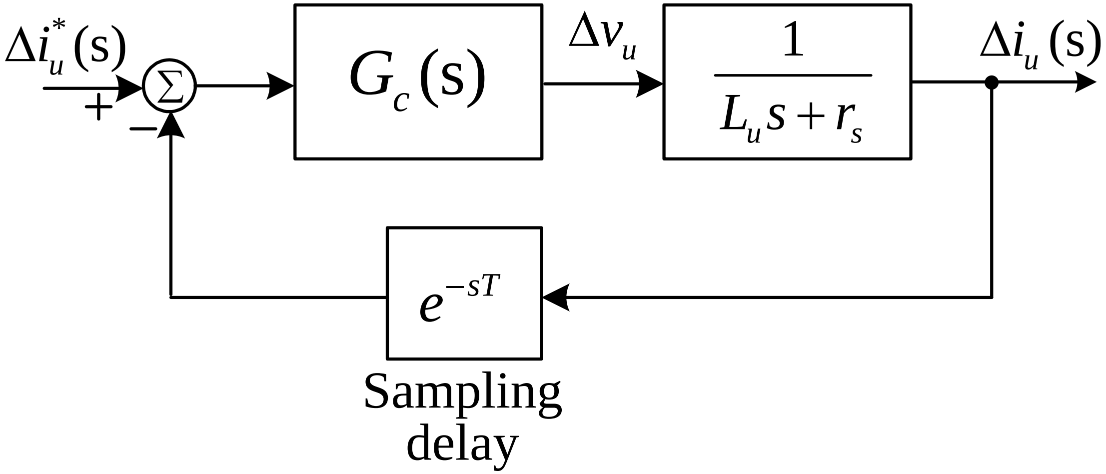
1. 控制架構
2. 電流迴路設計
3. **基於波德圖之電流迴路設計範例 - PI 補償器**
4. 電流迴路之驗證
5. 以示波器量測電氣功率及相關訊號
6. 結論

基於波德圖之電流迴路設計範例 - PI 補償器

□ 電流迴路之控制器設計

Controller specifications	
Crossover frequency	600Hz
Phase margin	52°

PMSM參數及規格	
參數	數值
L_d	14.26mH
L_q	14.80mH
L_s	14.53mH
R_s	1.74Ω
ke	$31.5V_{rms}/1000rpm$
額定轉速	2000rpm
額定轉矩	1.9Nm
額定電流	2.6A



$$G_c(s) = K_P + \frac{K_I}{s}$$

電流控制器採用比例積分形式

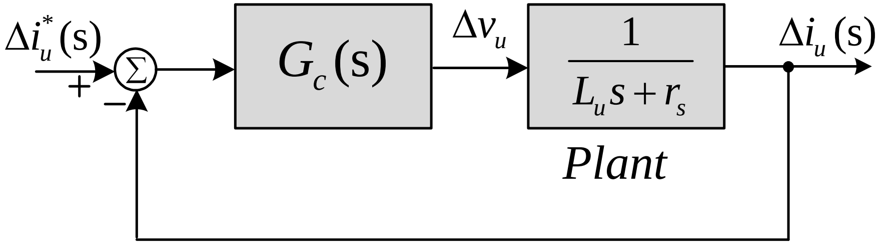
$$\omega = 2\pi \times 600$$

$$\phi_m = 52°$$

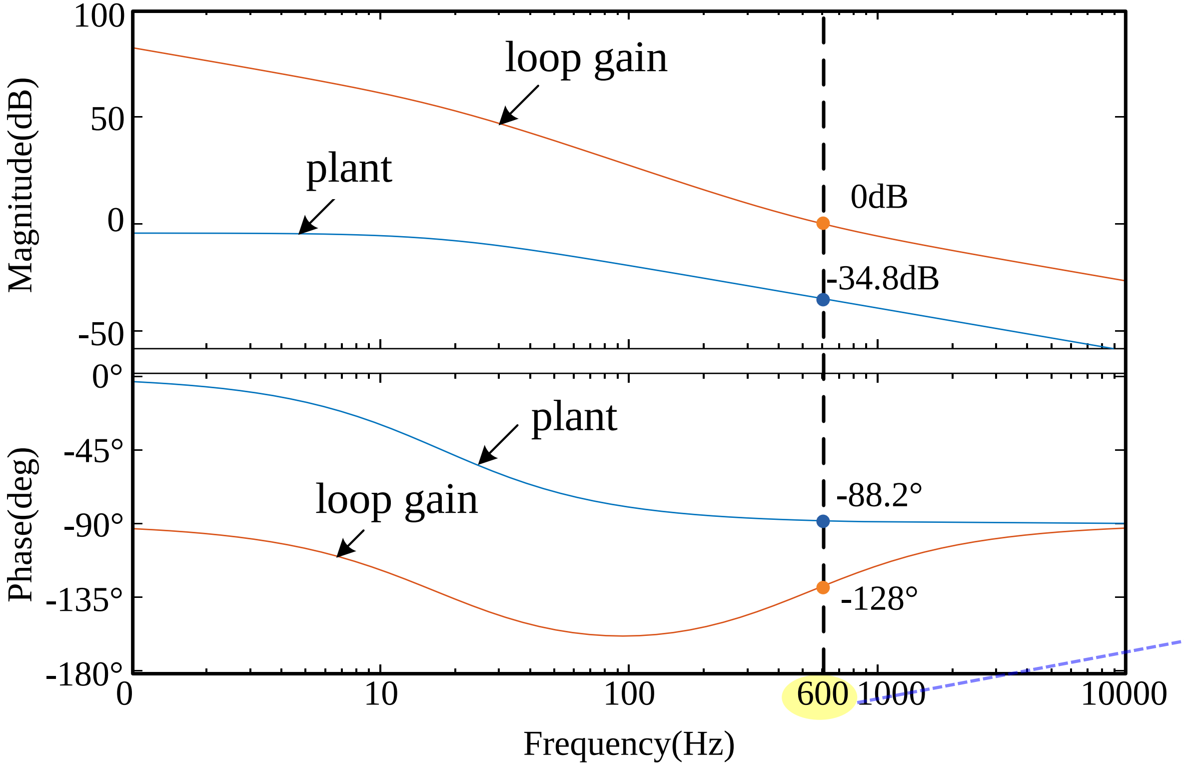
$$G_c(s) = 42.2 + \frac{13261}{s}$$

基於波德圖之電流迴路設計範例 - PI 補償器

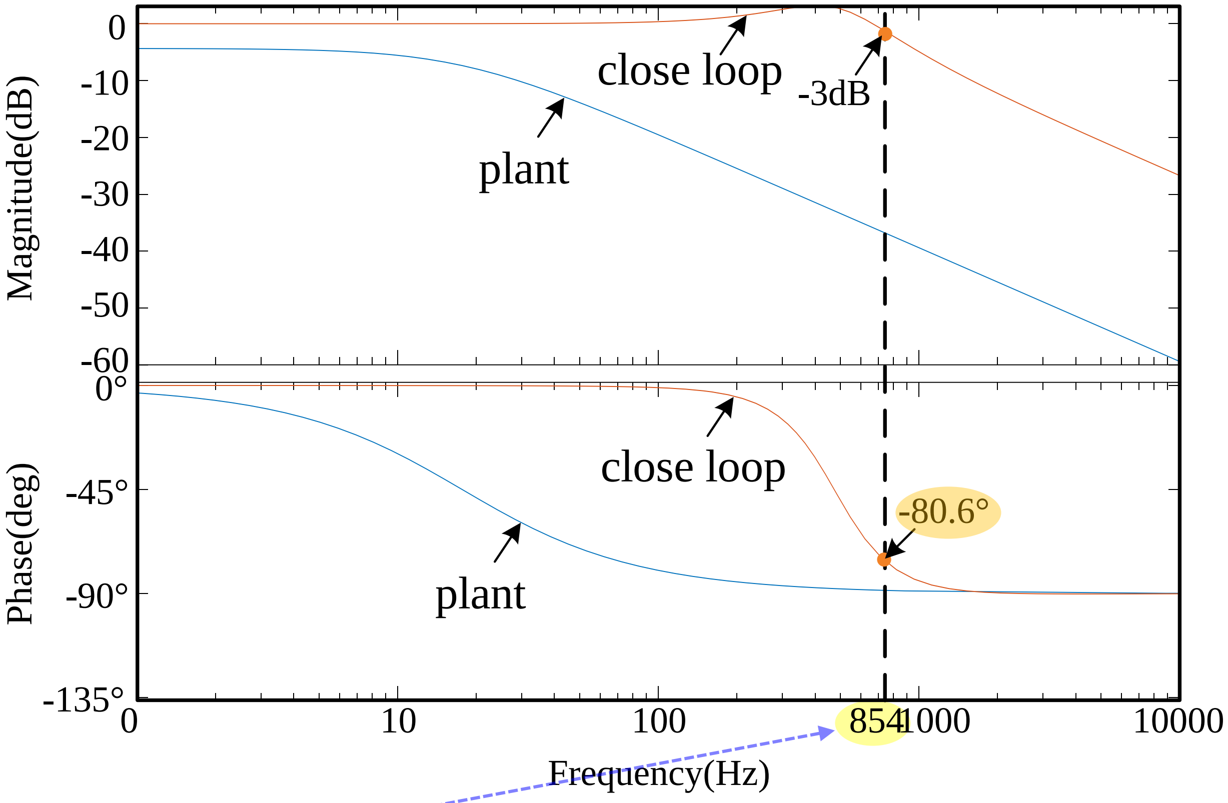
□ 電流迴路之控制器設計(Simulation)



Loop gain



Close loop gain



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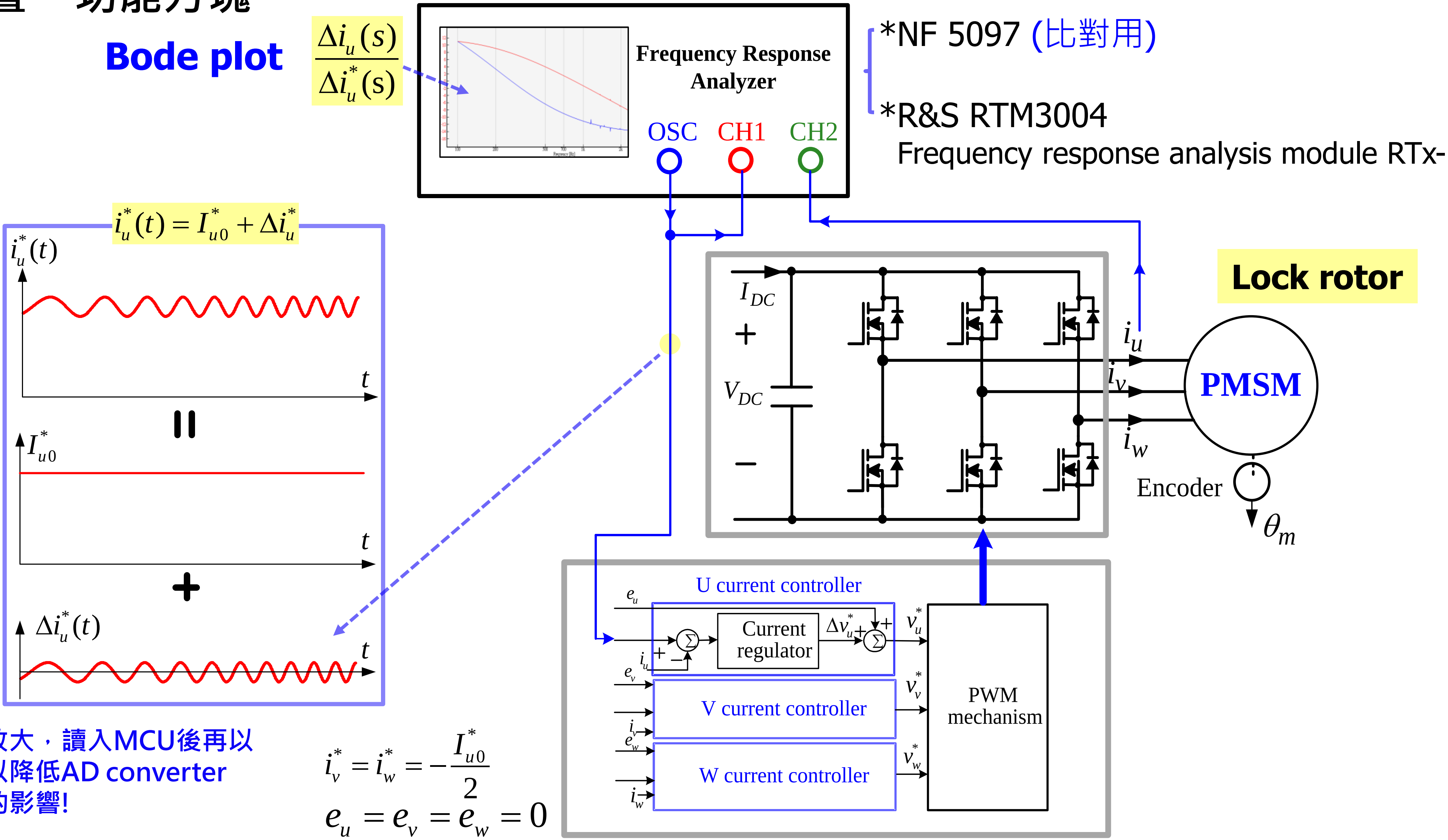
電流迴路之驗證

□ 驗證設置 – 功能方塊

Bode plot

$$\frac{\Delta i_u(s)}{\Delta i_u^*(s)}$$



Lock rotor

*NF 5097 (比對用)

*R&S RTM3004
Frequency response analysis module RTx-K36

工作點
(軟體產生)

擾動命令

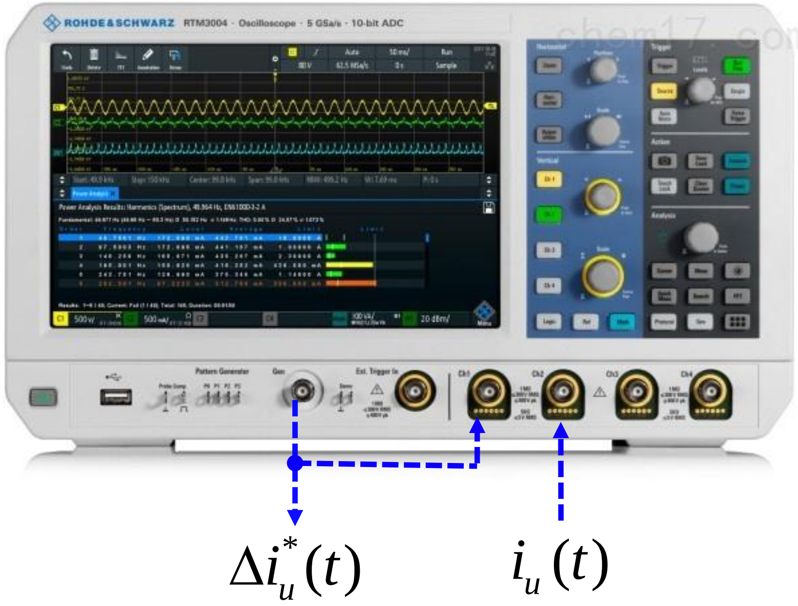
$$i_v^* = i_w^* = -\frac{I_{u0}^*}{2}$$

$$e_u = e_v = e_w = 0$$

輸入訊號先放大，讀入MCU後再以軟體縮小，以降低AD converter有限解析度的影響!

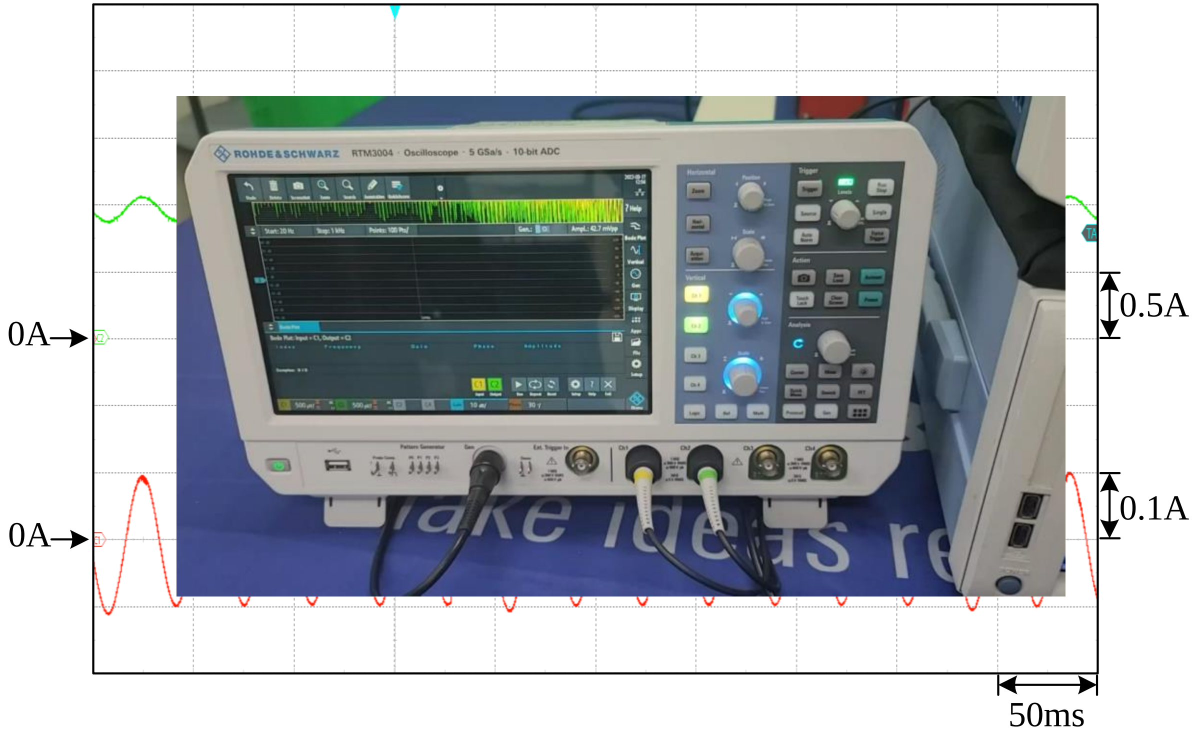
電流迴路之驗證

□ 驗證設置 RTM3004 with RTx-K36

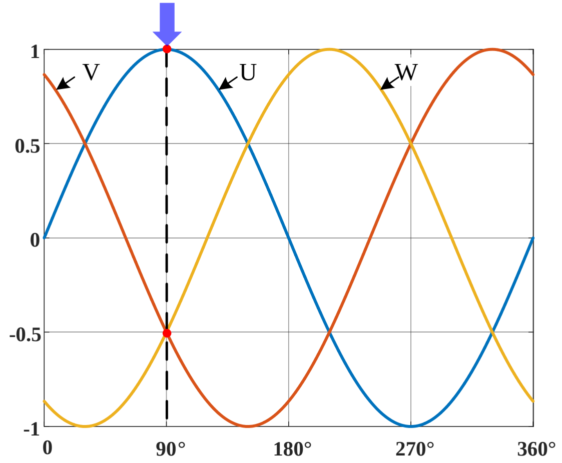


$$\overset{I_{u0}^*}{\downarrow}\qquad\overset{\Delta i_u^*}{\downarrow}$$

$$i_u^*(t) = 1 + 0.1\sin\omega t$$



將轉子角度固定於90°電氣角，分別將對應電流打入

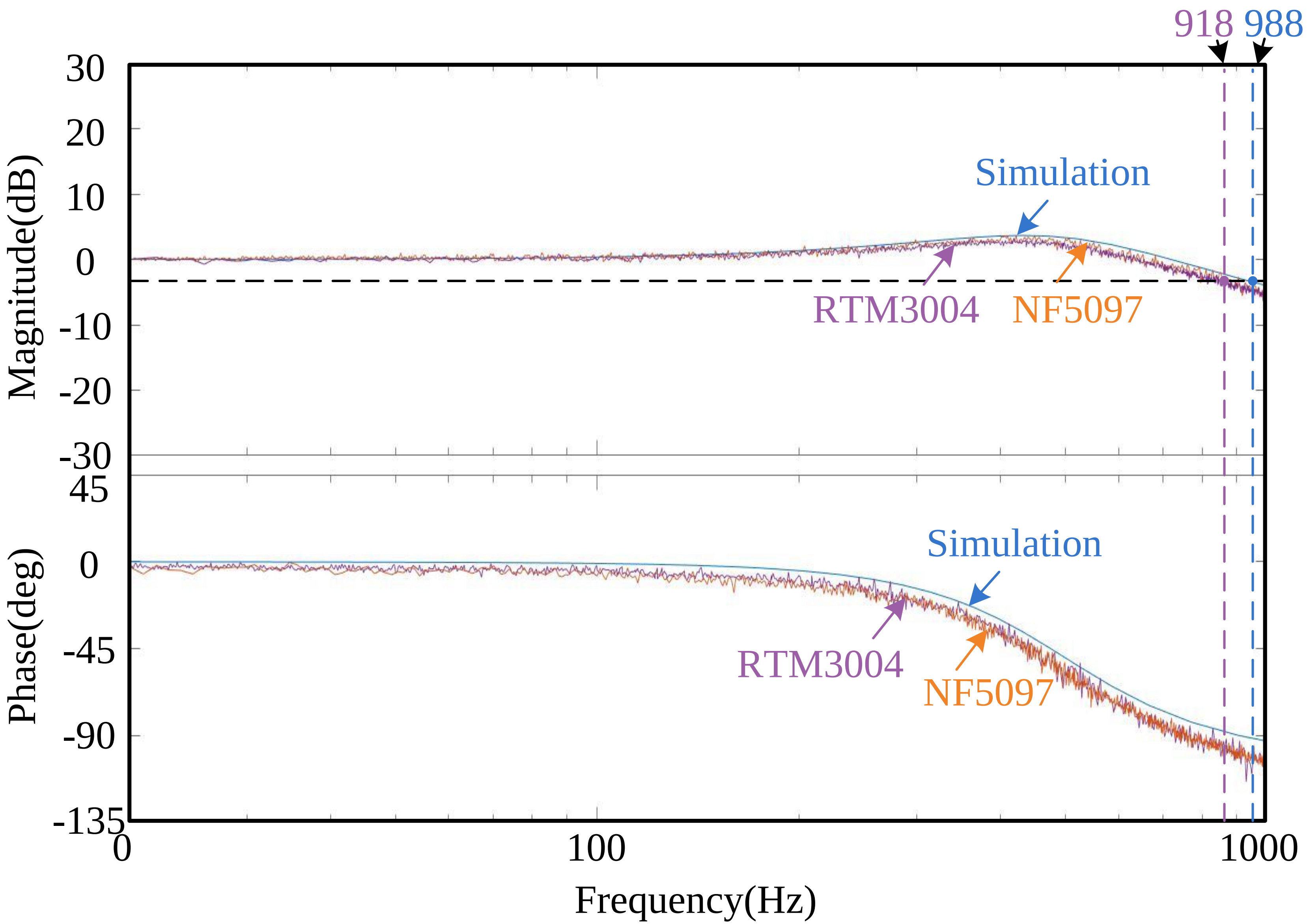


電流迴路之驗證

□ **測試結果 (RTM3004 with RTx-K36)**

✓ Bode plot

✓ Step response



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以示波器量測電氣功率及相關訊號

□ 輸入馬達電氣功率

- ✓ 有效功率(Effective power, P)

$$P = 3 \times \frac{1}{T}\int_0^T v_{uN} i_u dt$$

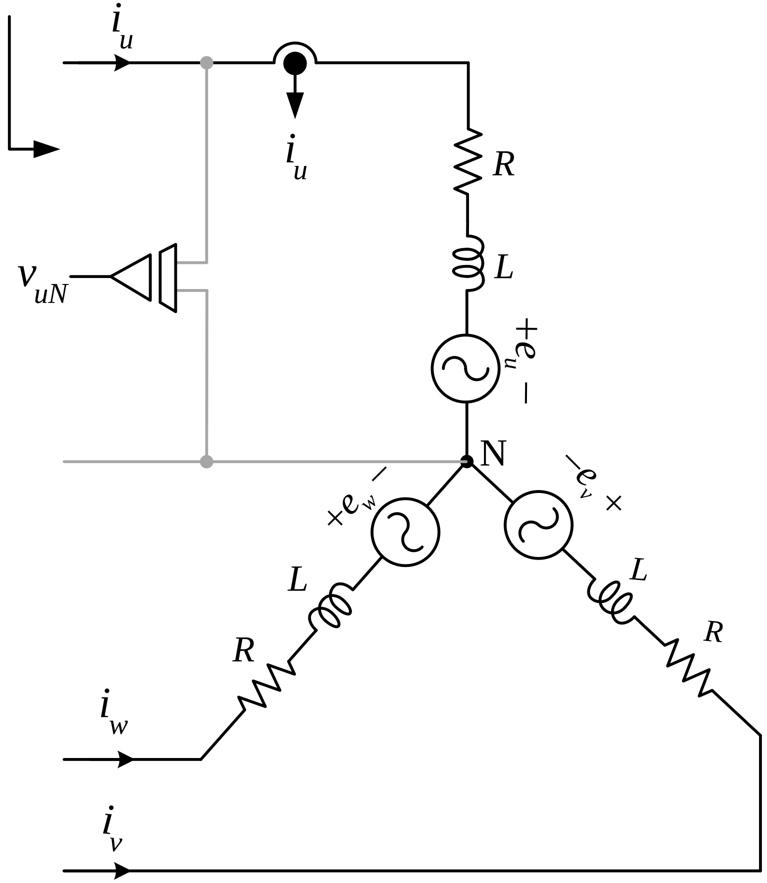
- ✓ 視在功率(Apparent power, S)

$$S = 3V_{uN,rms} I_{u,rms}$$

- ✓ 功率因數(Power factor)

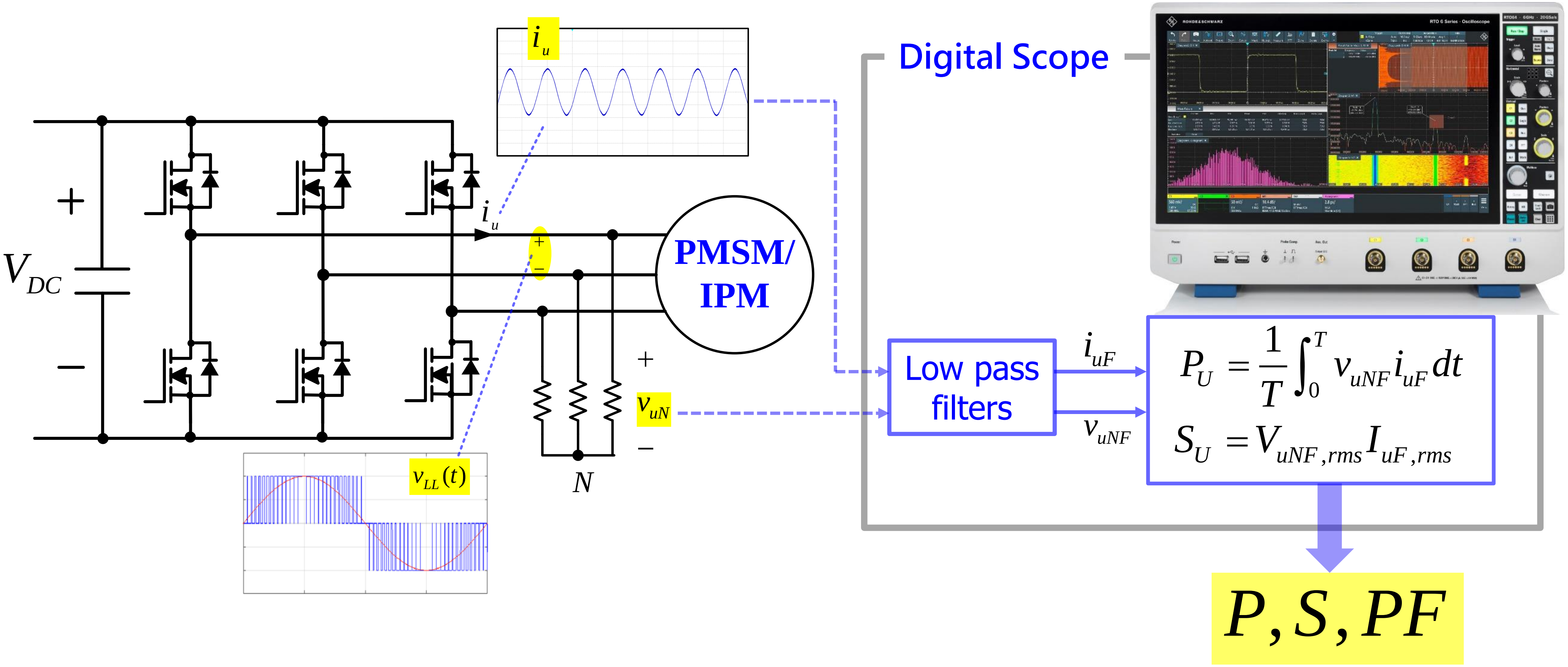
$$PF = \frac{P}{S} = \frac{\frac{3}{T}\int_0^T v_{uN} i_u d}{3V_{uN,rms} I_{u,rms}} = \frac{\frac{1}{T}\int_0^T v_{uN} i_u d}{V_{uN,rms} I_{u,rms}}$$

量測單相功率
(假設馬達平衡)



以示波器量測電氣功率及相關訊號

□ 輸入馬達電氣功率 - 示波器功能



Digital Scope

Low pass filters

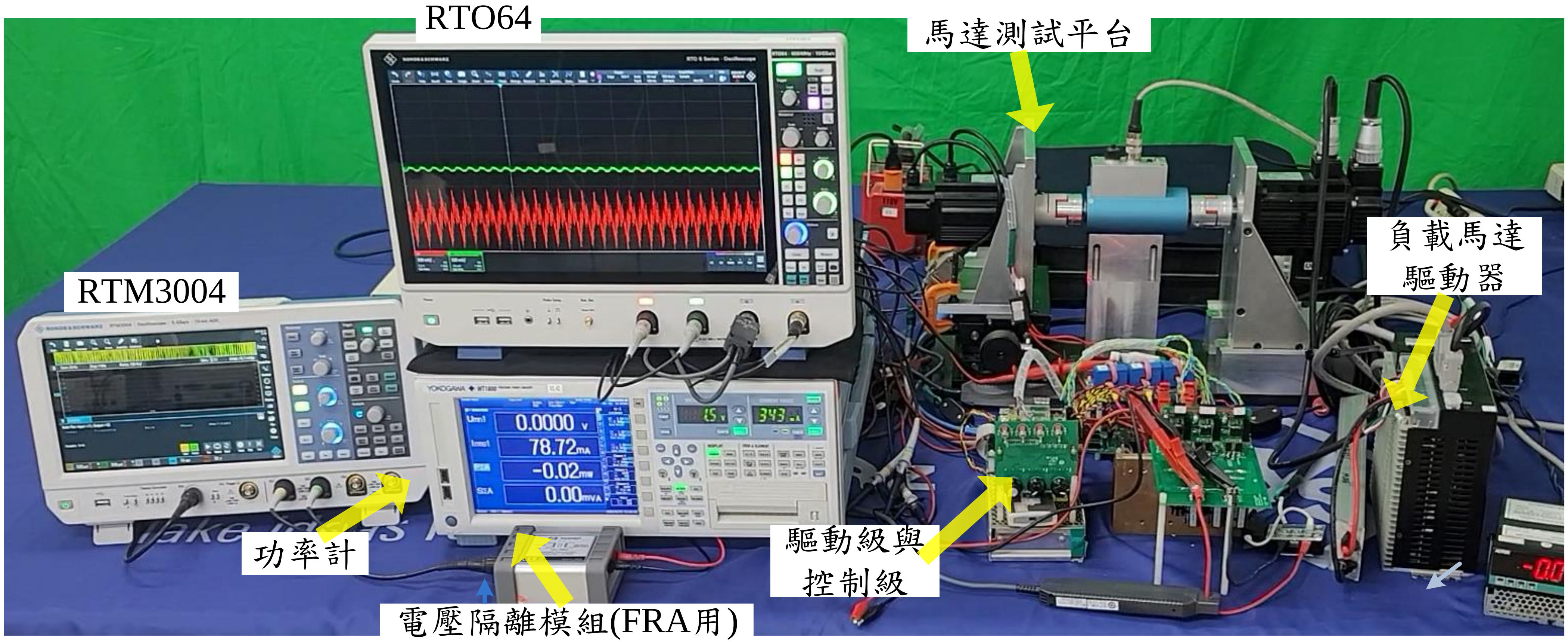
$$P_U = \frac{1}{T}\int_0^T v_{uNF}\, i_{uF}\, dt$$

$$S_U = V_{uNF,rms} I_{uF,rms}$$

P, S, PF

以示波器量測電氣功率及相關訊號

□ 量測設置及設備



□ 加載方式

- ✓ **Torque mode** : 原動機(負載)先將待測馬達帶到預定轉速，再由待測馬達輸出轉矩；
- ✓ Speed mode : 由待測馬達轉到預定轉速，再由原動機(負載)進行加載。

以示波器量測電氣功率及相關訊號

□ 量測設置及設備

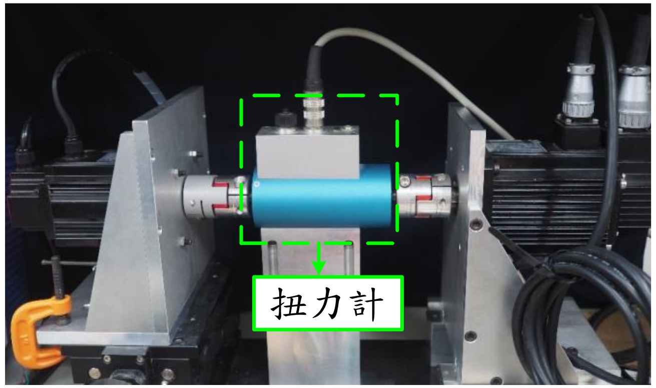
➢ 示波器(R&S RTO64)

- ✓ 600MHz ~ 6GHz bandwidth
- ✓ 1 million waveforms per second
- ✓ Sample rate up to 20 G sample/s
- ✓ 16 bits resolution (high definition mode)
- ✓ Fast Fourier transformation (FFT)

➢ 功率計(Yokogawa WT 1800) (比對用)

➢ 扭力計

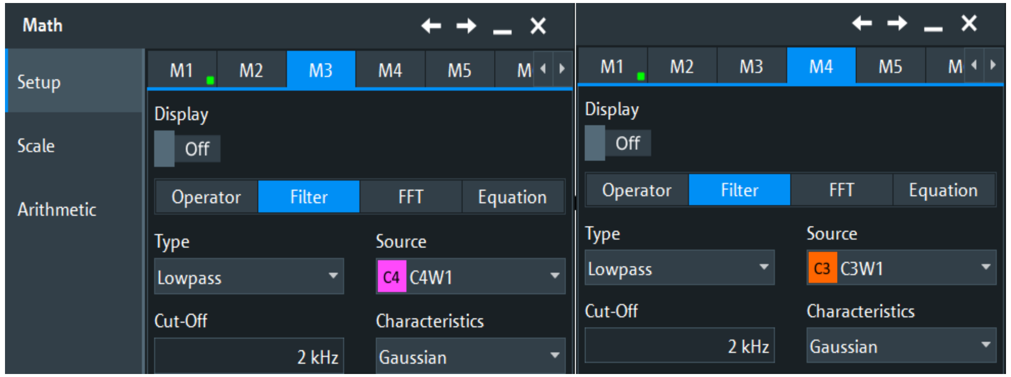
額定轉矩：10 Nm
額定轉速：20000 rpm



以示波器量測電氣功率及相關訊號

□ 量測設置及設備 - 示波器設定

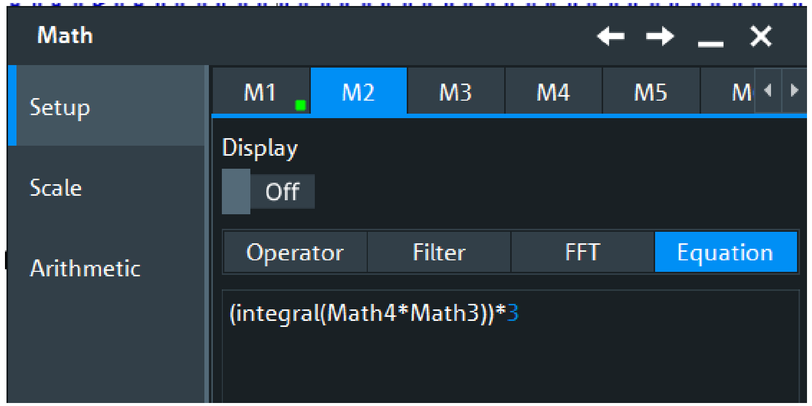
➢ 低通濾波器設置 (2 kHz)



M3: Display Off; Filter; Type Lowpass; Source C4 C4W1; Cut-Off 2 kHz; Characteristics Gaussian

M4: Display Off; Filter; Type Lowpass; Source C3 C3W1; Cut-Off 2 kHz; Characteristics Gaussian

➢ 功率量測公式



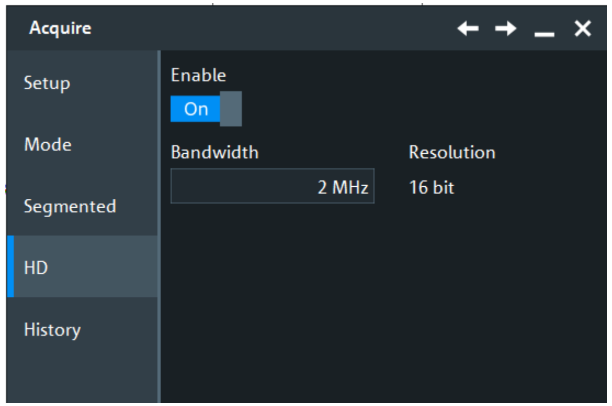
M2: Display Off; Equation

```
(integral(Math4*Math3))*3
```

➢ 快速傅立葉轉換設置

M1: Display On; Type Magnitude; Start frequency 0 Hz; Center frequency 665 Hz; Stop frequency 1.33 kHz; Frequency span 1.33 kHz; Resolution BW 1 Hz; Window type Blackman Harris; Frequency axis Linear; Span/RBW coupling Off

➢ HD 模式(高解析度)

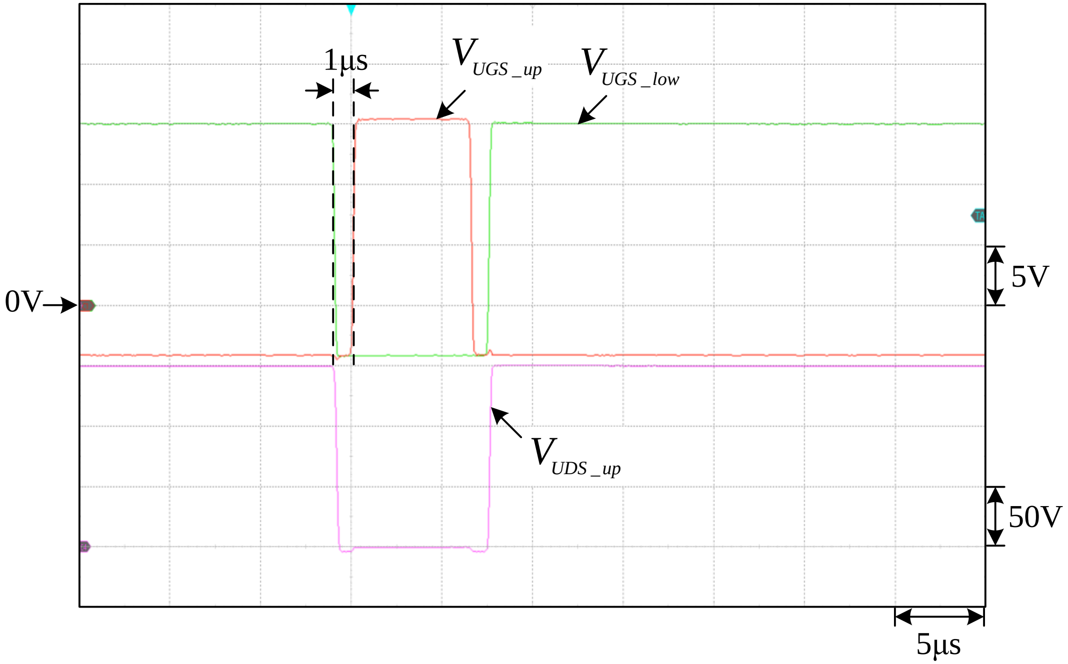


HD: Enable On; Bandwidth 2 MHz; Resolution 16 bit

以示波器量測電氣功率及相關訊號

□ 量測結果 – 功率晶體 dead time

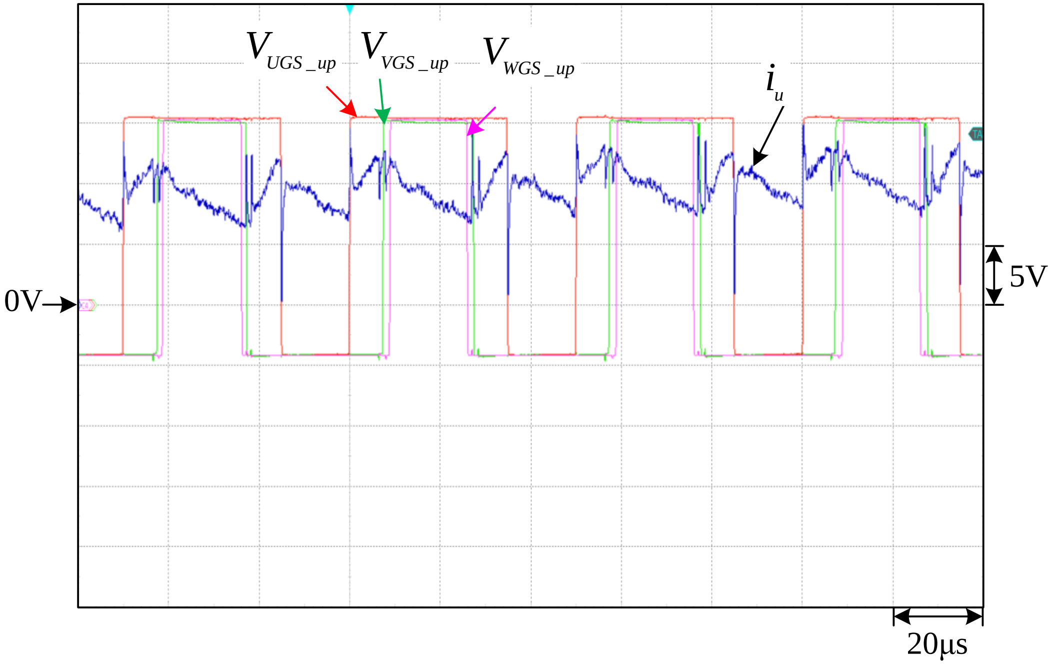
➢ 馬達U相開關



$t_{dead\ time}$=1μs

➢ 開關造成的電流漣波

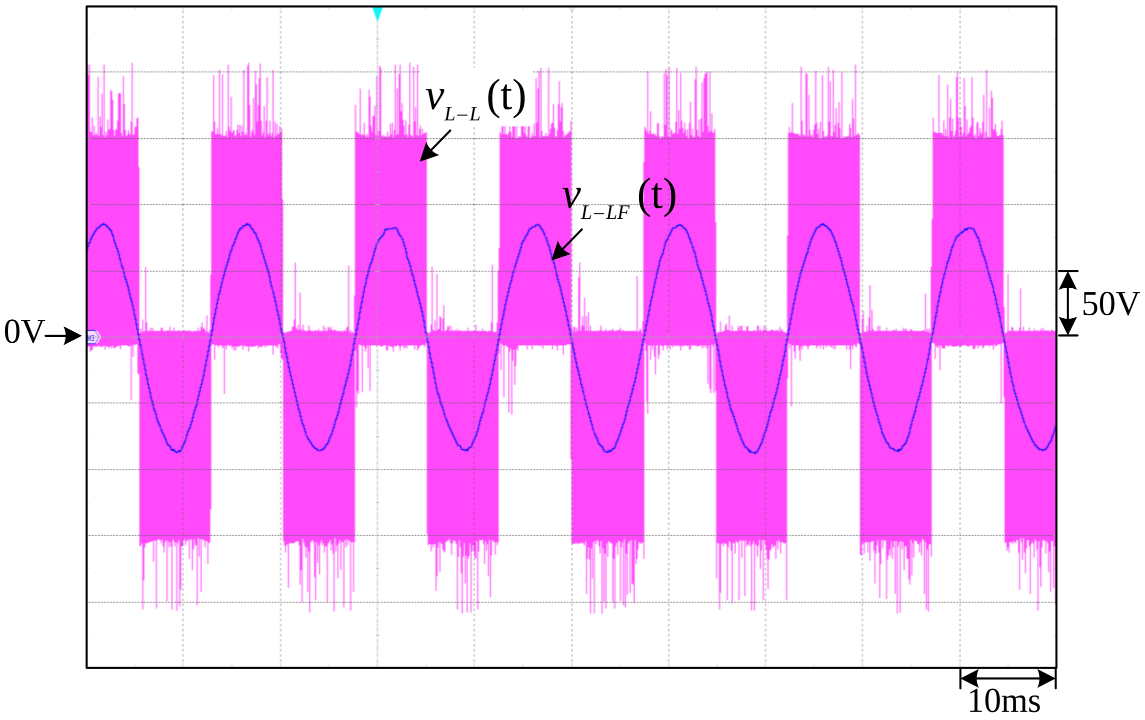
在開關切換時會造成線電流上出現電流漣波



以示波器量測電氣功率及相關訊號

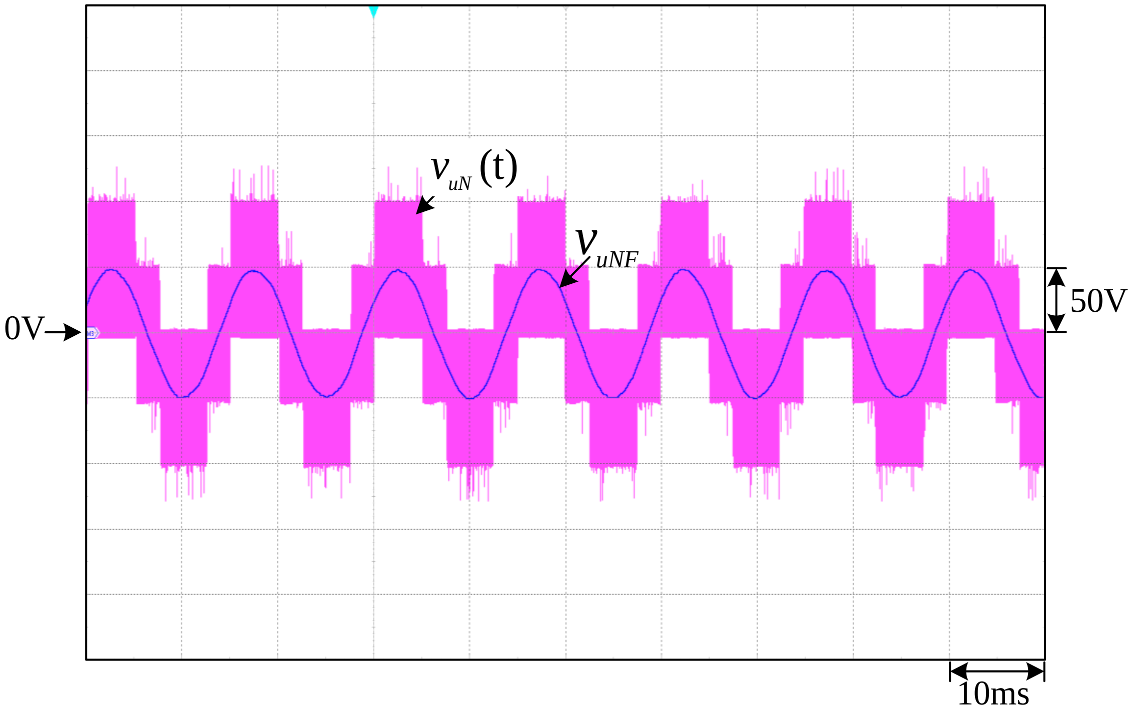
□ 測試結果 – SPWM調變下之馬達線/相電壓(@0.5Nm,1000rpm)

➢ 馬達線電壓



$$v_{L-LF,p} = 85.4V$$

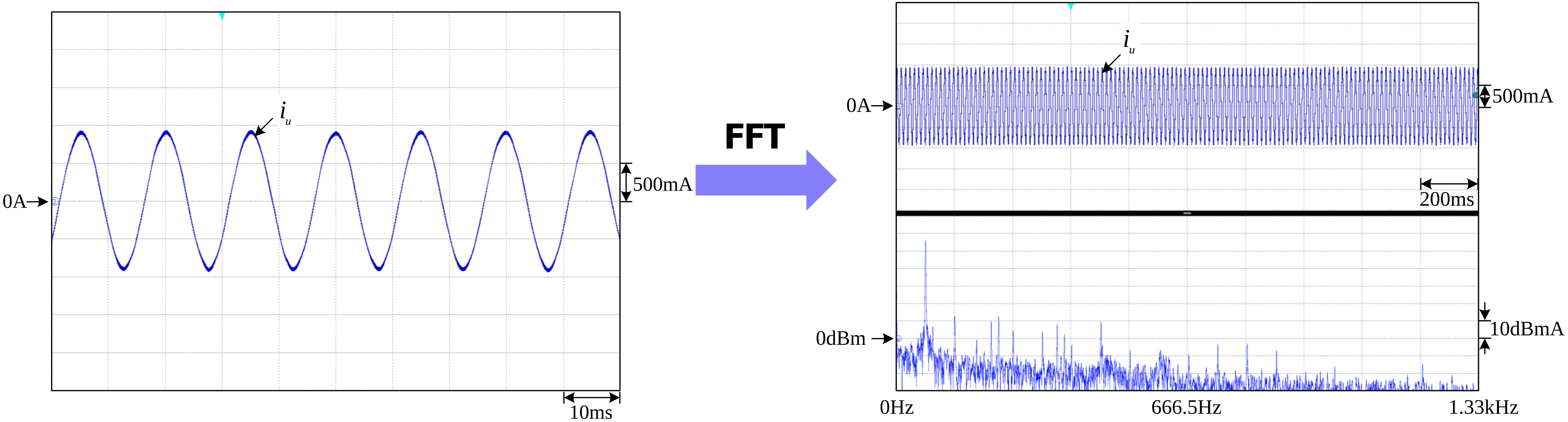
➢ 馬達相電壓



$$v_{uNF,p} = 48.9V$$

以示波器量測電氣功率及相關訊號

□ 測試結果 – 電流諧波(@0.5Nm,1000rpm)



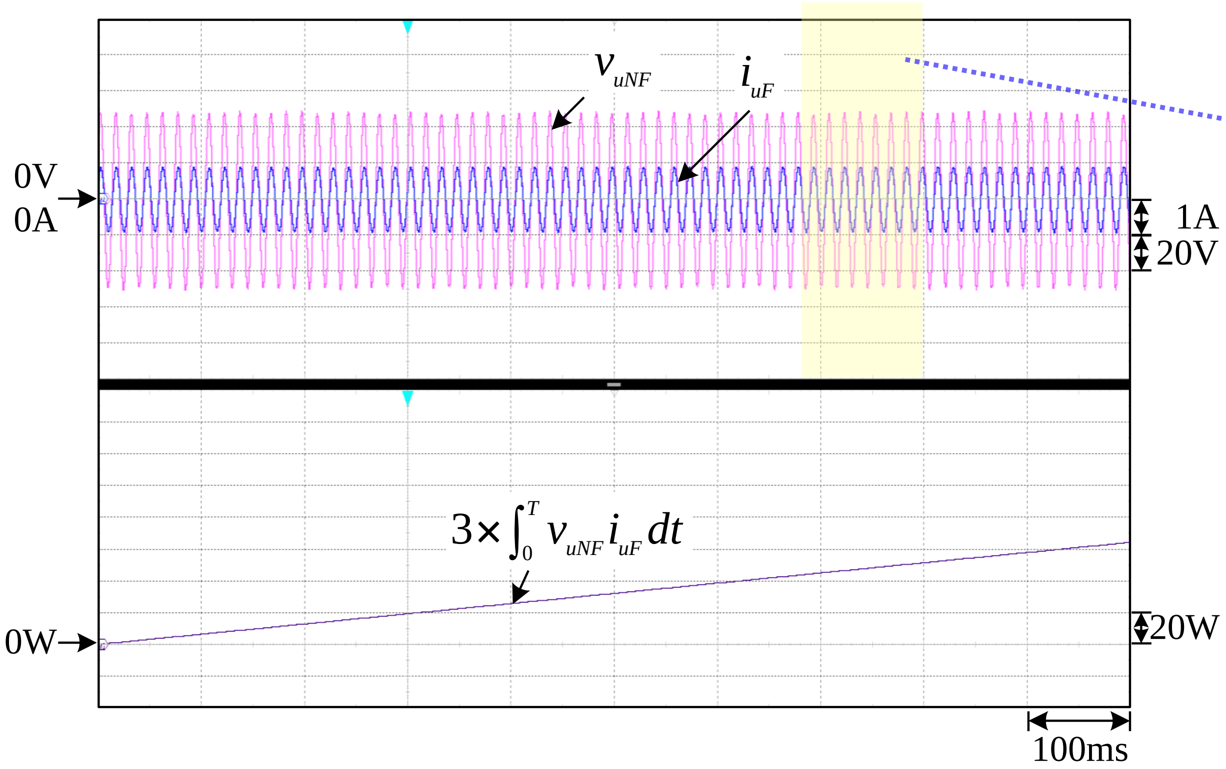
示波器FFT設定

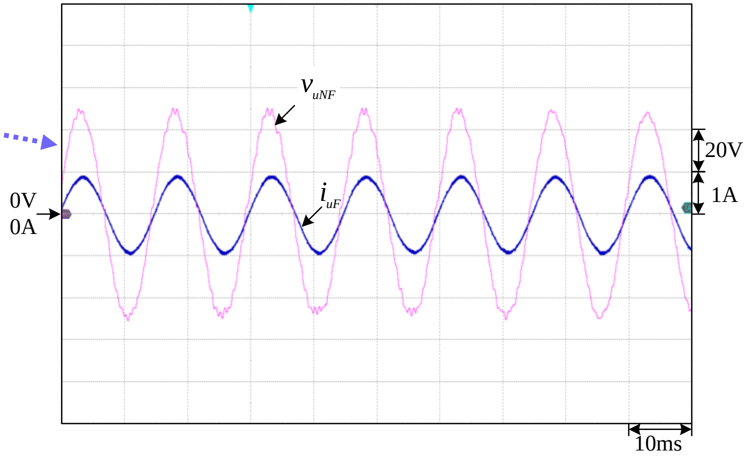
Resolution=1Hz
Frequency=0~1.33kHz
Window type：Blackman Harris

倍率	頻率(Hz)	大小(dBmA)
1	66.8	56.0
2	133.6	12.6
5	334.1	4.2
7	467.7	10.0
THD$_i$	0.01%	

以示波器量測電氣功率及相關訊號

□ 測試結果 – 功率量測(@0.5Nm,1000rpm)





示波器R&S RTO64量測值

$$P = 3\times\frac{1}{T}\int_0^T v_{uNF} i_{uF}\,dt = 63.04W$$

$$S = 3V_{uNF,rms} I_{uF,rms} = 65.18VA$$

$$PF = \frac{P}{S} = 0.97$$

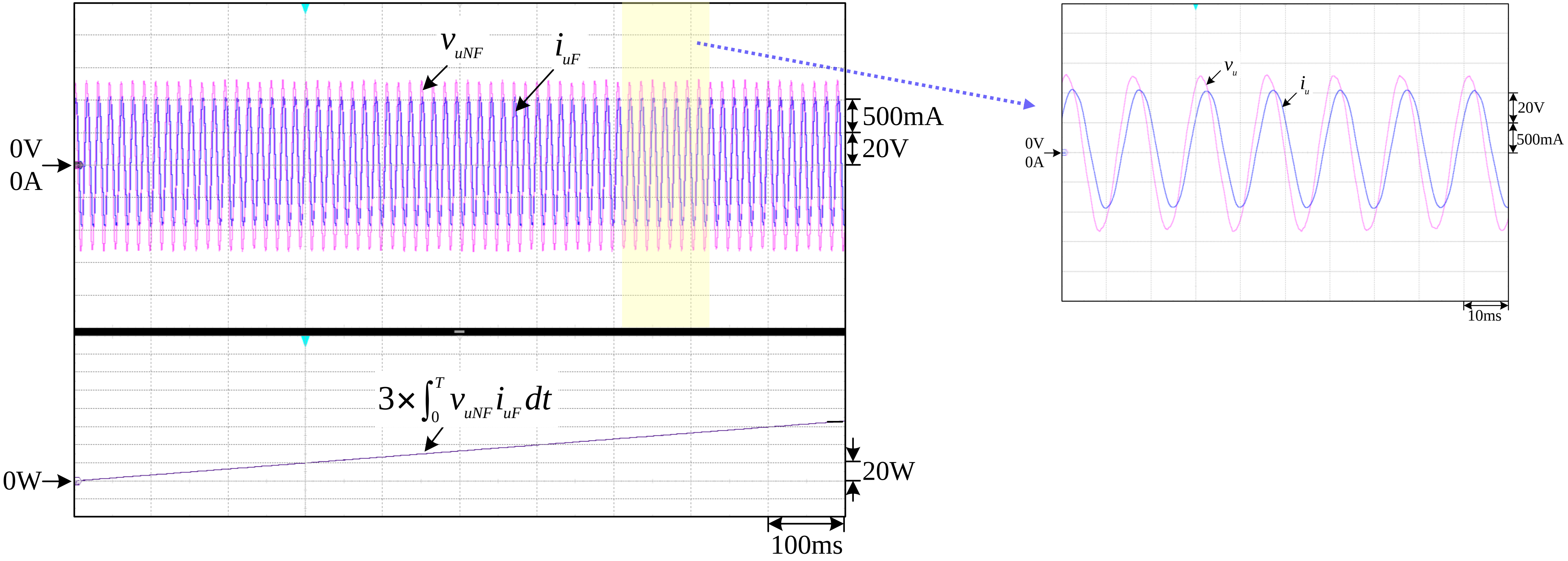
Power meter (WT1800)量測值

PΣA 64.600 W

SΣA 65.833 VA

以示波器量測電氣功率及相關訊號

□ 測試結果 – 功率量測(@0.5Nm,1000rpm)



示波器R&S RTO64量測值

$$P = 3\times\frac{1}{T}\int_0^T v_{uNF} i_{uF} dt = 64.60W$$

$$S = 3V_{uNF,rms} I_{uF,rms} = 78.0VA$$

$$PF = \frac{P}{S} = 0.83$$

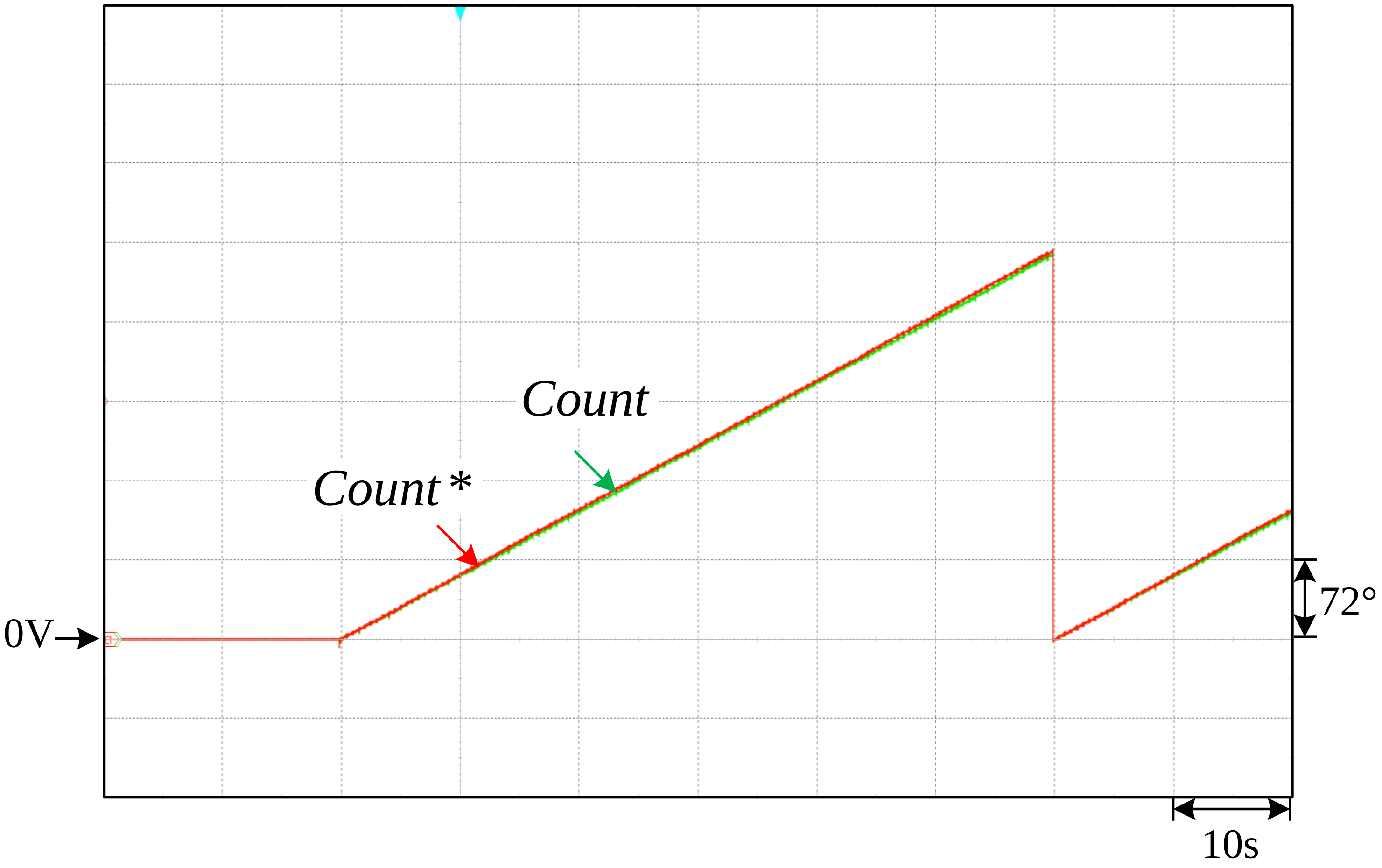
Power meter (WT1800)量測值

PΣA 65.904 W

SΣA 76.682 VA

以示波器量測電氣功率及相關訊號

□ 測試結果 – 1 rpm @無載



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4. 電流迴路之驗證
5. 以示波器量測電氣功率及相關訊號
6. **結論**

結論

- 馬達輸出轉矩特性與輸入馬達電流關係密確，在中低轉速下良好的電流響應可提較佳的轉矩輸出特性；
- 經由小訊號模型及**Bode plot**的設計程序，可讓電流迴路設計更具理論基礎；**Bode plot**設計與測試結果之趨勢相當接近，可推論所提設計程序的有效性；
- 傳統的永磁同步馬達及其驅動器的功能驗證多需使用功率計及頻率響應分析儀**(Frequency response analyzer, FRA)**等高階設備；近年來數位示波器的快速演進，可在合理的前提下取得馬達輸入功率及馬達驅動系統控制迴路之頻率響應資訊作為初步設計或驗證之參考。